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NAVAL RESEARCH LOGISTICS QUARTERLY

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SOME THOUGHTS ON LOGISTIC PLANNING FACTORS

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The singular feature about calculations for logistic support of military operations is that they are inevitable. No matter how difficult the process, or how long stalled, the requirement and capabilities for logistic support must sooner or later come down to hard, firm numbers expressed in meaningful categories of men, materials, and services. The tragedy of many an operation is that these numbers did not become known until it was too late to do anything about them. There is nothing chimerical about the realities of logistic support. They are always present, must always be dealt with, and will always affect the tactical and strategic operations of which they are an inseparable part.

Having accepted the thesis that some calculation is inevitable, the planner is then confronted with the problem of bringing the calculation process down to manageable proportions. This is the principal, indeed the only reason for planning factors. Planning factors are a sort of mathematical shorthand, a set of convenient abstractions of the mountainous detailed planning which would otherwise have to be done as part of the function of logistic planning. To this purpose, they are used, and misused, by planners all over the world in every phase of military activity. What follows here is an attempt to describe the nature of planning factors, to show what they are not, to attempt a description of what they are, and in general to advance some thoughts as to what can and what cannot be done with them.

REQUIREMENTS VS. CAPABILITIES

There is an important difference which is frequently missed between requirements factors and capabilities factors. So much of our work and discussion seems to focus upon the problem of determining requirements and we are perennially bogged down, it seems, on translating force tabs into tons of supplies and barrels of fuel and numbers of men. This is only half the field to be considered, however, since the limits of any logistic problem of any consequence are determined not by its requirements, but by its capabilities. This becomes even more important the higher up the planning ladder one goes, because in the last analysis we get just so much money, men, and material and they are not related, especially, to the job we may have to do. It is quite possible, in this shadowy, in-between, no-peace-no-war atmosphere that we are going to live in for the next five to fifty years that perhaps the most important element in our planning is the number of dollars we get and what we can buy with them; in other words, a dollar capability. Let it be repeated: the field of capabilities planning factors is separate from the field of requirements planning factors. It is just as big and complex, and just as important. Planning factors are just as much the rate at which airplanes and engines come off production lines, the ability of shipyards and repair ships to accomplish repairs, of seabee battalions to put up bases, of ports to handle cargo, or supply depots to store, receive and issue material, as they are the consumption of fuel by ships and planes, and like indices of requirements.

ITEMS VS. SYSTEMS

Secondly, it is of great importance that we learn to think in terms of the essential difference between items and systems, as they relate both to requirements and to capabilities. For example, take the difference between item requirements and system requirements. An operation will require so many thousands of barrels of oil, aviation gasoline, and jet fuel. It will chew up so many airplanes, pilots, bombs, rounds of 8" ammunition. The personnel concerned will eat up so many pounds of beef and flour and wear out so many pairs of shoes and use so many rolls of toilet paper. These are all item requirements. They represent a final expenditure of material and personnel at the point of use. But in order to get these items to the point of use, which may be thousands of miles away from their point of origin, we have to have a distribution system, and back of that a procurement system, and every single man, every barrel of oil, and every last bean and bullet makes a demand upon those systems for procurement, storage, movement and issue to the final consumer. So item requirements are irrevocably linked to system requirements, like two sides to the same coin.

Now it is out of this very matter of whether we are talking items or talking systems that so much confusion and misunderstanding have developed. When we talk about items we are talking about particular things—shoes, JP-5 fuel, boiler compound, 8" manila line, wiping rags. We are talking in terms of pairs, gallons, pounds, coils, and bales, depending on the issue unit of the items involved. The amounts by which these things are measured are non-additive, and would be even if the issue units were all the same. And there is implied the necessity for eventually computing, at some level, the requirements for literally millions of different kinds of things.

All this means that at the levels where people have to work out quick feasibility checks for specific operations (namely at the theater levels and below) there are going to be only a relatively few items that we can profitably use planning factors on. The people just are not available to sit down and work out separate factors on every item of supply they might need, in the first place, and it is not worthwhile, in the second place. There is a need to know at all times where one stands on fuel, major use items of ammunition, provisions, and a limited number of critical stores and repair parts; but generally speaking, the use of planning factors is limited to far less than one percent of the number of items in the Navy Supply System and that is generally about the level where we can expect it to stay.

When we talk about systems, however, we are talking about only a few broad categories of requirements, and we are concerned with only two basic units of measurement—weight and cube. The requirements of any one category are additive, so that we can add tonnages of dry provisions, repair parts, general stores and clothing and come up with a significant total requirement for the distribution system in terms of measurement tons of dry hold cargo demands upon the system. Moreover, there is great room for shifting overages and shortages between different items in the same shipping category, so that what we lose on the onions we can make up on the potatoes. So when we are considering systems, we do not have to concern ourselves with items as such, but only as they make a demand upon the system. We have to know what the items will displace in the following shipping categories:

Category	Unit of Measurement
Bulk Liquid Black	Bbls
Bulk Liquid Clean	Bbls
Dry Hold Cargo	M/T
Refrigerated Cargo	M/T
Ammunition	S/T or meas. tons
Deck Cargo	Sq. ft. or meas. tons
Personnel	Spaces

Even here there is a certain amount of flexibility. Dry cargo can be loaded into refrigerated spaces, hold cargo can ride on deck, ammunition can be shipped in any dry cargo vessel if necessity demands.

It is in this business of estimating system requirements (and capabilities) that planning factors can be extremely useful. It should be emphasized that ocean shipping is only a part of the entire distribution system, and at times it is not even the critical part. Many readers may recall that at the very time when shipping on the West Coast was at its most critical point back in 1944 and 1945 there were anywhere from one to two hundred ships swinging around the hook in various Pacific Atolls—retentions—because at the terminal they could not be unloaded, either through lack of stevedore help, or lighterage, or beach-head handling equipment, or trucks to move the cargo off the beaches. Some of the ships were retained simply because there was no place to put the material ashore—forward area commanders were using the ships for floating storage in the forward area. Whatever the trouble was, it meant that somebody, perhaps in his preoccupation with items had missed his guess either on system requirements or system capabilities, or both. And it emphasizes the importance that any planning factor which describes the operation of a whole system must not exceed the capacity of the narrowest bottleneck in it. For example, a distribution system for any one of the six general cargo categories must of necessity include all the following operations:

1. Continental storage and handling
2. Movement to tidewater
   - Air, truck, rail
3. Loading out
   - Labor capacity
   - Berth capacity (including intransit storage)
4. Overseas movement
   - Surface, air
5. Overseas terminal capacity
   - Labor
   - Berth and lighterage
   - Port and beach clearance
   - Storage and materials handling

INITIAL MOVEMENT VS. RESUPPLY

Thirdly, there is an important difference between planning factors for an initial movement and those for resupply. They are used differently and produce different results. Take the matter of item aggregation, that is, the control of a large number of different kinds of items by single planning factor. The Advanced Base Functional Component is a splendid example of item aggregation. A D-1 component means specifically so many quonset huts, so

many 6X6 trucks, jeeps, so many chief storekeepers, and if a fleet commander orders a D-1 he will get all these things and thousands more in the quantities prescribed in the outfitting list. The ASO allowance lists are another example of item aggregations.

Now item aggregations can be used freely in planning for an initial movement and by it the fleet commander can save a tremendous amount of detailed calculation. It is a one-shot proposition, and after the initial movement has been made the people concerned can gradually work down their overages and excesses and requisition shortages to bring their on-board supplies and personnel to a desired level. But we cannot use item aggregations for resupply, except in very special circumstances without borrowing trouble at compound interest. An example is the way in which we sometimes used the Basic Boxed Base Load (BBB) during World War II. This was a unit shipment consisting of about 7,500 items of GSM in "balanced" quantities for the initial stocking of an advanced base supply depot. After the first group of ships was serviced it was not balanced any more, and the depot's stocks never did get balanced until it was able to accumulate its own usage data and set up its resupply on a requisition basis. However, some depot commanders instead of going to an individual item requisition basis to resupply their needs, would merely send in a wire for another BBB, and out it would come, and then they would have ten years supply of what they only had five years of before it arrived—and their short supply items were still in short supply. They were using the BBB as a planning factor for item resupply, not for initial movement as it was always intended, and in the repetitive process of resupply the old errors were compounded every time a new shipment came in.

There are exceptions. Aggregations can be used for resupply if

1. There exists reliable usage data with a strong measure of central tendency on the items in the aggregation;

and

2. Usage can be rigidly controlled;

and

3. Liberal use can be made of substitute items.

Rations conform to these criteria, and to a less extent so do clothing and ship's store stock. We can, if we want to, put these three categories on an aggregate basis, and tie them up in neat packages of so many man-days supply and label them with a distinctive designator, and use them for planning factors. Up to a point, we can also substitute in certain categories of general stores items, but the possibilities for substitution in repair parts is practically nil. Machines are not nearly as accommodating as human beings in that respect. The possibilities for item aggregation on a resupply basis seem to find their practical limits in the three categories most closely related to human consumption: provisions, ship's store stock, and clothing.

DESCRIPTION OF LOGISTIC PLANNING FACTORS

Logistic planning factors are artificial values used to project, under specified conditions, the future requirements or capabilities of a given organizational unit in terms of men, material or services. They describe both systems and items, including aggregations of items. They may be employed for planning either initial movements or resupply. The ultimate purpose of logistic planning factors is to establish, with small effort and with good probability, the quantitative relationships between requirements and capabilities.

Good planning factors, correctly used, can perform a highly valuable service in helping to solve logistic problems. Their value lies in the fact that the planner using them can, with a minimum of time and effort arrive at approximately the numbers he would otherwise have to

obtain by an enormously complicated series of detailed calculations. And they perform this absolutely vital service: The planner can, with planning factors, determine reliably when a plan is definitely infeasible, so that he is permitted at the outset, before he has wasted his time and effort, to junk the plans that have no possibility of success. Then, having eliminated the courses of action that are definitely infeasible, the planners can further examine the retained courses of action to find out which of those promises to be the more rewarding, and which must be given further detailed treatment. Then, for each of the most rewarding courses of action, he is usually able to determine where the major deficiencies and trouble spots are which demand early corrective measures.

ORGANIZATIONAL BASES FOR PLANNING FACTORS

Planning factors rest upon two bases. One is organization; the other is the activities that organization carries on. If one disregards horses and mules, he can truthfully say that the final consumer of everything produced for the Armed Forces is either a man or a piece of equipment. Everything we get is either eaten up or worn out or used up maintaining a man, or it is burned up, shot up, installed in or on or otherwise used up maintaining a piece of equipment. Similarly the capabilities of the logistic support forces are ultimately based upon men and equipment. At its root, the organizational basis of all logistic computations, including logistic planning factors, is population—population of men and population of equipments.

Now obtaining good population figures of men is easy. We make an administrative habit of keeping track of our people and we always know pretty well where they are. So we take the organizational unit of man and make it the basic unit of measurement of all the supplies and services which are predominantly dependent upon population of men; i.e., consumption of clothing, provisions, and ship's store stock, medical supplies, hospital beds, so that we have requirements for and capabilities of these things expressed in terms of personnel spaces, man hours, man days, man years, etc.

But when we turn to the matter of trying to keep track of equipment populations we immediately run into all kinds of trouble because there are thousands of different kinds of equipments and in view of the mobility of our forces it would be impossible to keep track of them as individual pieces. Furthermore, when we consider item requirements for resupply, for example, we find that there are many kinds of general supplies, as well as repair parts, which are common to many different kinds of equipment. So it is not practicable except in the case of certain large and unusual pieces of equipment, to maintain population records of equipment types as such. And in lieu of basing our planning factors on individual types of equipment, we base them on aggregations of equipment, realizing that we are compromising, in the name of practicability, the principle that planning factors should be based upon population of the ultimate consuming or producing units.

As a result of this compromise, we in the Navy base our planning factors on the ship, the airplane, and the advanced base functional component as the most practical and logical aggregations of equipment. We even carry it one step farther and use them as the planning basis for aggregations of men as well, so that in logistics reference data one finds for example, the requirements for ship's store stock, clothing and small stores, and provisions stated in terms of tons per ship per month. This is entirely logical, and makes it easy to relate the strategic plan to the logistic plan, because our force tabs are all expressed in terms of ships, airplanes, and advanced base functional components. But we must never forget in our preoccupation with these aggregations of equipment and men that a carrier consumes a certain

amount of fuel oil a day not because of the shape of its hull, but because installed in that hull there is a main propulsion plant of so many thousand horsepower that is operated at a certain average speed for a certain number of hours a day. One of the major problems of developing and maintaining logistic planning factors is the constant search for really relevant organizational bases which at the same time are readily accessible.

ACTIVITY BASES OF PLANNING FACTORS

Leaving the discussion of organizational bases of planning factors for a bit we may give some attention to the other basis, that is, the activities that the organizations perform. Requirements are created, and satisfied, by men and equipment engaging in certain activities, and since we have found it convenient to group equipment types into ships, planes and functional components we can say that requirements and capabilities are the product of the organizational units of men, ships, planes, and advanced base functional components carrying on certain activities. How much the requirements and capabilities amount to depends upon:

1. The number and type of organizational units involved,
2. The scope and intensity of their activities.

Now we have no trouble with the first element—that is, the organizational units. Their number is controlled by those who plan the operation, so that it is a known quantity. It is fixed, and it does not change unless we know about it. But of the second element—that of activity, we have under the best of conditions only a partial control and only a partial understanding—because our activity is conditioned not only by what we elect to do, but by what the enemy compels us to do, and by what he does to us. So we have two basic kinds of activity that the organizational units engage in which are largely independent of each other.

First, there is the activity of routine operations or functions. Everything goes according to the plan of the day. The ship steams at a controlled speed which is the free choice of the Captain or the Admiral; the sailors eat three square meals a day, and routinely wear out clothes, buy candy at the ship's store and routinely get sick. The guns are fired according to a carefully worked out training schedule.

Secondly, there is the activity that results from combat operations with the enemy. The ship steams at whatever speeds for whatever lengths of time necessary to catch, maintain contact, or get away from the enemy. Gunfire and strike plans set only a partial pattern for ammunition expenditure; enemy aircraft and submarines do the rest. And the factor of battle damage enters the problem as regards both equipment and personnel.

Now is there any way that we can relate these wild, erratic activity rates which are so far beyond our control to a definite set of numbers that can be used as planning factors? We can do it—to a degree. If we have access to enough reliable data to give us a truly representative sample, we can, by carefully noting—and recording—the conditions under which a given organizational unit consumed or produced a measured unit of logistic support, establish a planning factor which will be representative of that unit under those conditions. For example, let the black oil factor for CVA's be represented by the number 0177, based upon the assumption that the ship will be underway 63% of its time and will steam at an average of 18 knots. This says nothing about battle losses, or engagements with the enemy, or sustained operations, and in that alone lies its chief virtue. It admits that the planning factor is significant only under the standard conditions described—and it puts the commander on notice that he must, through exercise of judgment, modify the factor if the conditions under which he wants to use it are different from those given as standard. It is this recognition that planning factors are controlled

by a specific set of conditions, that they do not have universal application, and that professional judgment is always required that is the really significant contribution that the George Washington University Logistics Research Project has made to the philosophy of logistic planning factors, and it is the one thing fundamentally new and constructive that has happened in that field since the armed forces began to use planning factors. In other words, the notion is now established that planning factors can be set up for certain organizational units engaged in certain types of activity under a set of specified conditions. Then, as the commander's estimate of the situation discloses the differences between the standard conditions and those which are expected to exist in the proposed operation, the planning factors may be scaled up or down as dictated by his judgment.

RELIABILITY OF PLANNING FACTORS

This is easier said than done. We actually have to fulfill two separate conditions of reliability. First, the so-called "standard" planning factors have to be good or there is no use going further. But then, even assuming good standard factors, we have to have a reliable method of scaling those factors to adequately express the difference between the standard conditions and those which the estimate of the situation brings out. In other words, we have to have more judgment and less Jesus in the J-factor. Planning factors after all are predicted values, not actual values, and they are usually, but not always, based upon some sort of average of past performance. They do not produce results that can be predetermined on a strictly causal basis, and about the best that one can possibly hope for out of a planning factor is that it will give a reasonably good chance that the actual requirement or capability will not fall excessively above or below what it was predicted to be. Whether we are figuring requirements or capabilities, or systems or items, or initial movements or resupply, there is an element which bears directly upon the reliability of the planning factor: namely, the dispersion of the actual usage value about the predicted value of the planning factor. It goes without saying that the narrower the dispersion— that is, the closer the dots are to the norm— then the more reliable the planning factor X is. This is important because in planning we can never deal with the whole. We can only deal with infinitely small pieces of the whole— samples, in the language of the statistician. Planning factors are based on samples of past performance, and they are applied to predict the behavior of other samples of future performance. It is important for the future prognosis that the mean value taken as the factor of the past performance is truly representative of the sample.

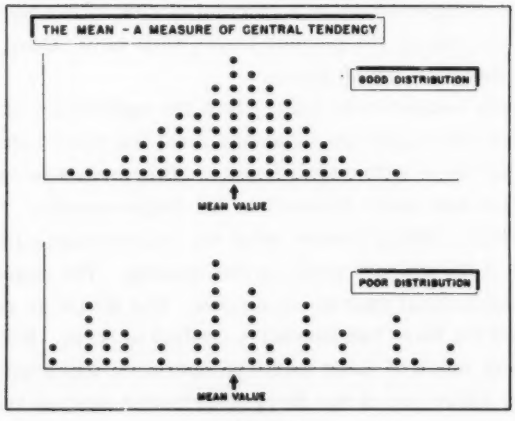

Figure 1

For example, the chart at the top of Figure 1 shows good central tendency, and the probability that the planning factor X would be reliable even if it were derived from a sample of only half the number of dots shown here. The bottom chart shows a poor central tendency, with the commensurate poor possibility that any sort of decent planning factor can be gotten out of it. Of course, assuming each of these dots represent the consumption of a certain ship for a given item, the planner could conceivably wind up applying the factor X against the four dots on the high extreme end of the scale in the top chart. But the probability is overwhelming that he would by any random choice pick up four dots that were fairly close to the mean value, so that his planning factors would still be reasonably reliable. But in the case of the bottom chart, his chances of any four dots lying close to the mean would be pretty small.

There are other statistical safeguards that need be mentioned. The sample used to base the planning factor on should be fully representative of the type of activity it is set up to describe. The Navy engages in a number of general types of war operations: carrier task force operations, amphibious assaults, convoy and escort, ASW operations, and underway logistic support force operations. Then, of course, there is the routine maintenance of naval forces in the theater which always accounts for a substantial portion of support. These different types of operations differ radically in the demands they make upon the logistic support system, and we should probably have a set of planning factors for each of them. The availability of ship types will have its effect on how good a sample is, which is why, generally speaking, that we are going to have much better planning factors for ship's parts of a Class 692 destroyer, of which we have some 150 in service than we ever will for the three Class 41 CVA's. There is simply more data available on destroyers because there are more of them.

So much for the standard factors. The difficult second part to the problem—that is, how one scales the standard factors to accommodate the difference in conditions between the standard and the particular, is one for which there are as yet no really good answers. Referring back to our factor of 0177 for black oil consumption of a CVA on a 63% underway assumption at 18 knots, one can immediately see the difficulties involved in a situation where the commander wishes to run his force at 22 knots, to be underway continuously for a month. How to go about scaling the difference between the standard and the assumed ones? Fuel consumption produces some strange looking curves. The importance of this judgment factor, which deserves more attention perhaps than any other part of the factor, can scarcely be emphasized enough. An excellent set of standard factors can be absolutely ruined by a ham-handed application of the judgment factor in the same way a carefully drawn tactical plan may be ruined by an ill-considered judgment masquerading as a calculated risk. Like the risk, the judgment factor may be calculated, but unless the planner knows what he is doing, it may be nothing more than a blind, stupid, emotion-ridden guess.

A few generalizations remain to be made about the application of planning factors at different levels. One is that the higher the command level the more concerned the planner is with capabilities and the less he is with requirements. There are several reasons for this. At lower levels, the capabilities are better known than the requirements. They are easier to figure. The commander knows, within reason, what his logistic capabilities are—he knows how much oil he has and where it is, and how much he has coming. The same thing applies to ammunition, provisions, and critical spares—to a point. The things he requires are already in the distribution system, and the Navy has executive control of them. He may generally have some trouble calculating how much of these items an operation will require. But at departmental levels we are at the bitter end of our Navy distribution system and we are face to face

with that vast, nebulous jumble of capabilities, the American Economy. We do not have control of that, and there is no way in the world of determining with any degree of assurance the industrial feasibility of a war mobilization program in peacetime. The production complex is too big and complex to try to get down to cases, which fact contributes to the validity of the assertion that at the defense level it is much easier to figure the requirements than to determine the capabilities for meeting them.

Another generalization is that the usefulness of planning factors focuses upon the theater, fleet and service force levels, and tapers off rapidly in both directions. Perhaps the lowest point of usefulness of planning factors would be the task force level, and undoubtedly, many of the factors would not apply even then.

A third generalization, and this goes back to the samples discussed—planning factors have distinct applications at different levels. An Army division slice is 20,000 men at the division level, 40,000 men at the theater level, 60,000 at the defense level. A theater maintenance factor of so many tons of supplies per man per month is not a reliable factor at the station level.

Summing up, planning factors may be described as

. . . artificial values used to project, under specified conditions, the future requirements or capabilities of a given organizational unit in terms of men, material or services. They describe both systems and items, including aggregations of items. They may be employed for planning either initial movements or resupply. The ultimate purpose of logistic planning factors is to establish, with small effort and with good probability, the quantitative relationships between requirements and capabilities.

To which might be added:

Planning factors are most useful for planning at major field command levels.

Planning factors are based upon military organizations and the scope and activity those organizations carry on.

Planning factors are reliable to the extent that

1. The observations of the data on which they are based have been accurately recorded in accordance with recognized statistical methods.
2. The pattern of the observations shows good central tendency.
3. The conditions under which the planning factor is used can be quantitatively related to those under which it was developed.

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TRENDS IN LOGISTICS[1]

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One reads in Churchill's memoirs his observation that, "Past experience carries with its advantages the drawback that things never happen the same way again."

On the other hand, many logistic lessons of the past must be retained as fundamental. We must keep in mind the need for basic resources upon which a logistic-mobilization structure can be built and the principle that strategy and logistics are completely interdependent. Therefore, to understand new trends in logistics, we must consider new trends in strategy, tactics, and the weapons about which both are shaped. It is well to remember that no plan, either logistic or strategic, is of any value if it is beyond the capabilities of the nation. Also that operations, both logistic and strategic, must not leave the nation impoverished in manpower and materials lest the war be lost while winning the campaign.

Yet as Emerson wrote, "The world rolls; the circumstances vary every hour." For example, the 1946 edition of Encyclopedia Britanica has an article on uranium that says, "Uranium's chief use at the present time is in the ceramics industry." Philosophers seem generally to agree that there is no certainty but that of change—that all things are processes.

In that light, yet keeping in mind the lessons of the past, let us consider new trends in logistics under four main points, which will be discussed one at a time:

1. Logistic frugality
2. Speed and accuracy
3. Development of indigenous resources
4. Logistic problems in the light of new weapons.

From our reading we know that in the past there have been plateaus in the military art, times when the advances in weapons proceeded at a slow and evolutionary rate. But we are not in an evolutionary period now. The striking power of weapons is increasing by mutation in a revolutionary way. The science of destruction has advanced more in the last 10 years than it did during the preceding centuries following the invention of gunpowder. Today we could deliver more firepower in the first 30 days of a war than has been expended in all previous warfare to date. And yet our skill and daring in combat will avail us little if we neglect the logistic problems or fail in understanding the logistic trends which, while based upon the lessons of the past, now emerge under the pressure of a revolution in armament.

LOGISTIC FRUGALITY

All of us take pride in American productivity. Our free production and consumption between wars have allowed a relatively lush standard of living. Our cars are big, our refrigerators are big, like our deep freezers and television sets. When we get a new set of tires, we just throw away the old tubes. They are still usable, but they are cheap and there are plenty where those came from. Mass production and free enterprise in a land of great natural resource have provided abundance.

[1] This paper has been prepared from lecture notes on this topic written by various members of the Armed Forces Staff College.

But our wartime production is recalled with greatest pride. It has been referred to as our secret weapon. In two world wars we have overwhelmed our enemy to no small extent by sheer weight and force of material. As a matter of fact, some staffs literally overwhelmed our own troops with weight of material. General Eisenhower's theater historian [12] is quoted as saying that too much weight on the backs of our troops caused more deaths on Omaha Beach than enemy fire.

In any event, we know that overloading a soldier can cut down his chances for survival—unless he is smart enough to throw the stuff away—which he usually is. If he happens to need the stuff sometime, it has been part of our American way of life to know there is more where that came from. We backed up every American soldier in Europe with 46,000 pounds of supplies, and in the Pacific with 36,000 pounds [1]. During World War II we produced enough rifles and carbines to allow two for each of our eight million soldiers [10]. During World War II we produced enough small arms ammunition to shoot every man, woman, and child on the face of the earth sixteen times—after giving each of them two atabrine tablets [4].

So we are justly proud of our productivity. With 9% of the world's population and 8% of its area, we produce almost half of the world's goods and services [5]. However, we are in danger of running out of materials. We can not escape the indictment that as a nation we have been spendthrift, that we have neglected conservation—reclamation—maintenance. Our industry calls for five trillion pounds of materials a year, and for years we have been skimming the cream off our resources in unimaginable amounts. We wasted enough natural gas in 1950 to meet the needs of 11,000,000 homes. More sulfur goes up in industrial smoke stacks than is consumed. Half the petroleum in a pool is left behind in "production" [13].

In any event, our great peacetime consumption, a couple of world wars, and the present defense effort have gone far in transforming us from a "have not" nation in materials upon which a logistic-mobilization structure is built. In fact, that part of our way of life which holds, "There is more where that came from," is fallacious. The logistic barrel is not infinite. It is just so big. It has a bottom, and the bottom is already being reached in some critical instances.

I have mentioned these points to establish them as causes, and to indicate that frugality will be the effect; to indicate that frugality will no longer merely be a virtue to be reluctantly discussed by commanders at Information and Education meetings, but that it must be the mode of operating. At the height of World War II nearly half of our nation's production was being used to support our Armed Forces and those of our allies [6]. While our living standards remained generally high, there were shortages in the civilian economy. Just how much tighter the civilian could have drawn his belt and maintained our impressive production is conjectural.

But we do know that we fought World War II with an undamaged industrial machine, and that in a future war we will suffer damage within the continental United States. Some of our critical war production—and the extent is unknown—will have to be diverted to the rebuilding of production facilities. It is our hope to survive that war and to retain our freedom and some of our riches. That hope, in the face of the probable nature of the war, establishes frugality as a trend in logistics.

Even in the event that we are not forced into a full-scale war, by virtue of an attack on us or our friends, we know we will have to attain and maintain a sizable deterrent force for an indefinite period. Our people are able, and it appears they are willing, to build and maintain the right kind of force. But we can be sure our people will not sustain a force that is wasteful in either its composition or its management. Without the confidence of our people our whole

venture toward security would fail. Extravagance and mismanagement on our part during this difficult semipeacetime could destroy their confidence almost as surely as would our defeat in war.

The proof of the trend toward frugality has ample evidence on the bulletin boards of the three Services which reflect the cost-consciousness campaign. There is evidence of the trend in the fact that major commanders have been called to Washington for training and conference on the subject. There is evidence of the trend in our functional type budgets, carefully watched over by comptrollers; or again in the fact that all Services are attempting to boost combat strength while decreasing total manpower. That tightening of the manpower belt typifies the new trend in logistics—frugality—which springs from the logistic principle mentioned earlier: Operations, both strategic and logistic, must not leave the nation impoverished in manpower and materials—lest we lose the war while winning the campaign.

SPEED AND ACCURACY

In meeting the demands of future warfare logistics is more concerned with time than distance. Combat forces do not care how far away something is. What they want to know is whether they can get it when needed. All trends seem to point towards the attainment of speed, towards the attainment of accuracy and of economy. Speed, accuracy, and economy in distribution; speed and accuracy in the forecasting of requirements; speed and accuracy in feasibility testing; and speed and accuracy in answering the myriad and one questions that confront the logistic planner.

In order to achieve dependability in supply, we have been in the habit of stocking huge quantities all along our line of communication. For example, from D day in Normandy to VE day we discharged in Europe an average of about 75 pounds per man per day. Yet, throughout this same period the average daily consumption per man was only 40 pounds [2]. Almost twice as much was taken into the area as was consumed—the excesses going into reserves in the various depots.

Of course, we must maintain some safety level in the overseas area to insure against loss in transit and to permit a constant flow of necessary items. But, do we need even the more normal 60-day safety levels? Our supply agencies in an area of operations load and unload, transfer, ship and transship, store, and distribute about 90 days of supply to support our troops for 30 days [3]. And just what does a single day's supply amount to? Well, for a force of a million men this would be about 20,000 tons of supplies. For the construction of storage facilities alone for this one day of supply we would need the full effort of a construction battalion for one month [7].

An overseas commander may not find it necessary to stock such large reserves of all items, provided that he can be sure of their rapid replenishment. This change of thinking will come about only after confidence is felt in the ability of an improved system. All Services are now attempting to effect this improvement by strengthening the control of stocks in the overseas areas and by shortening the supply distribution cycle to make it more responsive to the current needs of the command.

The quantities of supplies in the overseas areas are now being reduced by bringing all stocks in the area under the requisitioning objective. This is an important improvement in stock control; it effects an economy in operation. At the same time it insures a better balance of stocks by considering all which are locally available and requisitioning only those items which are truly in short supply.

As the mechanization of all stock control procedures overseas is completed, we can change to a system of performing the computation of required quantities by machine methods. By a greater reliance on the most modern computers and other business machines, we are eliminating most of the man-hours previously required for this job. For example, the time required in computing and preparing requisitions in the Far East Command has been reduced from approximately 30 days to less than ten days [3].

Thus we see greater speed and accuracy are being obtained in the accounting and requisitioning processes. Furthermore, by increased reliance on electronic communications, by stream-lining processing procedures in the Zone of Interior (ZI), and by developing uniform methods for use in all depots, we are cutting even more time from our supply distribution cycle. Much of this progress, I repeat, we owe to the amazing capacity of our latest business machines. However, these machines, good as they are in obtaining speed and saving manpower, impose a requirement for well-trained personnel to operate and supplement them.

Now as to speed in transportation. Again, I emphasize, it is not a question of how far-away the supplies may be. Rather, we are concerned about how soon we can get them. Seeing this and knowing there is no place on earth more than 60 hours away from any other place [14], we can assume an over-growing demand for air transport and faster sea transportation.

Today, requisitions for supplies can be filed by radio, telephone, telegraph, or air mail. Today, fast trains, trucks, ships, and aircraft bring many supplies from the depot to the consumer. The advent of the cargo helicopter has increased the flexibility of aerial supply to our most forward and isolated units. Accordingly, some propose that practically all reserve stocks be kept in the Zone of the Interior and delivered by air directly, or as closely as possible, to the using unit. Backing this thought is the fact that a light 20 per cent of our supply, because of its technical nature, constitutes 80 per cent of the supply budget [2].

Such direct delivery to the using unit helps solve the waste and expense of the pipeline and obviates some of the difficulties of storage and distribution. But it would require an enormous air fleet, great skill for successful operation, and a vast expansion in airbase facilities, not to mention air supremacy. Flying overseas that light 20 per cent of expensive items is fine and might even save money. Air transport to furnish those "missing links" on emergency requisition is highly desirable. But we cannot economically transport all supply by air, particularly the millions of tons of petroleum, oils, and lubricants (POL) required overseas to support combat.

However, increased transportation speed alone does not greatly affect our over-all logistic picture. If everything could be transported overseas by air, we would save in the pipeline only the number of days supply equivalent to the difference between air and water transportation—say 30 days—regardless of the length of the war. In a war of similar duration to World War II this would amount to only slightly more than 2 per cent. Since it is not likely that the majority of supplies can practicably be carried by air, the over-all savings due to faster transportation alone become relatively insignificant.

The point I wish to make is that, by combining speed in transport of critical items with accuracy of forecasting and inventory, we can obtain a much greater degree of dependability in the flow of our supplies. With such dependability the need for these huge safety reserves at overseas depots will be greatly reduced. By progress along these lines we may well look forward to cutting our current overseas stockage objectives in half. This would be a significant improvement toward which all Services are now working. Depot construction could then be cut in half and depot operating personnel greatly reduced, while our supply cycle would actually be kept more closely in tune with the requirements of the combat forces.

DEVELOPMENT OF INDIGENOUS RESOURCES

I have listed the increased utilization of local resources as another trend. Actually, this may more properly be considered as a countermeasure against a longer-range trend of all wars to date. In each war our military forces have become more dependent upon their home base and line of communication for the essential technical supply that gives a modern force its punch. This, of course, has meant greater use of our limited materials—greater logistic support.

Obviously this tendency to rely more and more on continental U. S. must be retarded in order to conserve our materials, transportation, and manpower. While I do not advocate that we re-establish the foraging party as a primary means of subsisting our forces, there is unquestionably a definite trend toward obtaining more of what we need closer to the locale where the requirement exists. As a matter of fact, in World War II more than 25 per cent of the supplies for our forces in Europe were obtained overseas [11], and the "offshore" procurement activities have grown in magnitude since that period [9]. While much of the current motivation for this program is due to peacetime economic considerations, there is little question but that the procurement sources thus established should greatly enhance our area supply situation in wartime.

Economy in manpower utilization has become a major problem. All Services have been striving to reduce the number of military personnel which must be assigned to supporting roles. The trend toward greater use of indigenous labor in Korea is apparent from our recent operations [15]. We must plan for maximum utilization of local manpower and existing facilities in any overseas area. The savings in transportation requirements alone make this only too obvious.

Similarly, the development of natural resources as close to the scene of the requirement as possible will reduce much of the load on our supply system. One very good example is the local production of construction materials rather than the shipment of great tonnages of lumber, sheet metal, fittings, etc., from the United States. Give a thought, if you will, to the savings that could be effected by obtaining sufficient POL in the vicinity of our operating forces. Even if we must build refineries and develop additional sources, think of the magnitude of our military petroleum requirements and consider the effect of reducing the necessity for long-distance transportation of these products.

LOGISTIC PROBLEMS IN THE LIGHT OF NEW WEAPONS

Much has been said and written about the new developments and new weapons with respect to their effect upon strategy and tactics. We want to think about their implications in logistics and thereby see logistic trends. I do not have the futile purpose of trying to be prophetic. I only want to highlight trends and stimulate thought. Accordingly, let us project ourselves into the near future when we may reasonably expect the use of guided missiles of a modest 1,000-mile range; thermo-nuclear warheads measured in terms of megaton TNT equivalents; abundant atomic weapons of all types; aircraft capable of supersonic speeds; chemical agents of thorough deadliness; cheap biological agents of terrifying aspects; and submarines 50 times more difficult to defeat than those which in the past cost us so dearly.

To highlight trends, let us visualize the effects in the next 20 years of the new developments in hardware, strategy, and tactics on supply, medical service, transportation, construction, and management.

First, supply. I made some observations earlier to indicate that, as all of us know, there is a lot of it. The trend throughout the ages is that there will be ever-increasing amounts of supply in terms of tons per man. From the military point of view we can talk about supply in three phases—procurement, storage, and distribution.

First let us consider procurement. Unfortunately for the supply people, the taxpayers, and therefore all of us, most of the new items will be regulated items—complicated, highly technical, and very expensive. The new items generally will be manufactured in the Zone of the Interior and hard to obtain in the Communications Zone (Com Z). Local procurement of many of the new weapons will be out of the question. On the other hand, non-essentials such as extra clothing, furniture, beer, prefab buildings—furnished in lavish profusion during World War II—can be dispensed with or acquired locally in most areas. But even with proper emphasis on frugality and development of local resources it is doubtful if we will be able to pull tons per man down where it was.

Next, under supply considerations, is storage. This activity will be even more affected by new trends. Everything must be dispersed for protection against the new weapons. This presents a dandy problem to mull over. Some day it might be necessary to maintain balanced stocks in a number of areas. But you can imagine the cost, the waste, and the increase in supply, required by such a plan. Then what about local dispersion of storage? Consider that an ordnance depot in the last war with 150,000 tons of ammunition, occupied an average space of 10 square miles [7]. From this you can compute that in a future war that same depot will need about 1,000 square miles. This is because the area required varies as the square of the distance between stacks of ammunition. If you double the distance, you need four times the area. If you increase the distance ten times, as is now necessary for protection, you increase the area 100 times.

However, such dispersion is absurd. At that rate there would not have been enough space on the British Isles to have properly dispersed merely the material that was required to mount the invasion of France, and all of France would have been used to disperse the supply we had on hand there when we crossed the Rhine. That illustrates the magnitude of the problem. Storage underground will be utilized when possible; for example, when deep mines or natural caves exist. This will not help much, however, due to the difficulties of moving supplies in and out and providing lighting and ventilation. For example, gasoline in drums can not safely be stored in a closed space because of the danger from fumes and explosions.

Distribution and issue, the third facet of supply, also will be made more difficult by dispersion of supply, as well as by dispersion of the customers, the troops themselves. Again more transportation, time, and manpower will be required. More documentation and management will be essential to keep things straight; and quantities stocked in any area must be held to reasonable proportions.

Actually, in the face of the new weapons, the difficulties of storing and distributing supplies in the Com Z may be so considerable that we would not want to pile up great reserves there, as we used to do.

After supply the second logistic function to look at in the light of new weapons is evacuation and hospitalization. We are all aware that much progress has been made in fighting disease. World War II was the first war in history in which death from disease did not far exceed death from battle casualties [8]. But to maintain the trend, our medicos face three new problems arising from the armament revolution. First, new facilities and treatments must be developed to overcome or protect against the effects of radioactivity. Also, there will be the

sinister threat of chemical and biological agents. Together they will doubtless require much in the way of new supplies, technical skill, and vigilance. Finally, these three killers—atomic, biological, and chemical weapons—will require greatly increased care of the civil population in the Com Z. The bare minimum of care and treatment required to avoid interference by civilian casualties with military operations may reach tremendous proportions. The sanitation problem alone created by the sudden death of 100,000 people, such as occurred at Hiroshima, is very unpleasant, but one that must be dealt with if the area is to remain habitable.

A logistic service that will be most radically affected by new development is transportation. Ships and railroads are the carriers really moving the tonnage. Both, however, will provide good targets for the new weapons. Ports will be hit with atomic weapons. Therefore, plans must be made to make maximum use of beaches and artificial harbors. Large convoys, unless the interval between ships is increased, will be suitable targets for atomic weapons; and dispersed ships are lucrative prey for submarines. Similarly, railroad shops, yards, and bridges will receive their share of destructive attention in the Com Z.

Accordingly, it is not unrealistic to assume that aircraft will take a more reliable place in the transportation picture. We now have aircraft carrying 400 troops or 50 tons of cargo—the C-99. Nuclear power and other new power plants under development promise aircraft of greatly increased performance. Rocket assist could reduce the need for great take-off length on runways; and tracked gear, reverse rockets or props, the mat, and improved landing technique could reduce the need for great landing runways. But then, of course, large air facilities also become prime atom targets, and dispersion and defense become a nuisance. Be all that as it may, the trend is toward greater employment of air transport. True, it should be used only when justified and is economical only if utilized for costly or critically needed cargo. But it may definitely help to reduce the amount of many items that have to be stored in the area.

All this is fine, but there is a big fly in the ointment—the ointment and fly both being POL. About half of the trmendous tonnage taken overseas in World War II comprised gas and oil, moved in tankers [2]. That is big business—millions of tons even then. And the trend is ever upward—pushed up by war of movement in heavier machines, by dispersion and in turn concentration, by jet engines, and by the use of cargo aircraft to replace surface carriers in the Com Z. Yet, in view of the diminishing oil reserves [5], loss of oil by sinkings would be hard to take. If there were no reason other than POL, we had best keep the sea lanes open. But whatever we do, POL will remain a major logistic problem. Perhaps we may hope the Navy will expand its program of logistical submarines. Or perhaps we can develop POL sources and refining capacities closer to where the product is needed.

Construction is not a new service, but more of it will be required—a trend shown in every war to date. Rapid dispersal, and in turn concentration, will require more roads, bridges, and alternate routes. Since the entire Com Z and possibly the ZI will be subject to air and guided missile attack, repair requirements will soar. More airfields will be required, and these must support 100-ton or heavier aircraft. Underground shelters will be very popular and will be asked for headquarters, hospitals, assembly plants, and launching sites, among other uses. They will have to be large, deep, and rapidly built. With large ports atomized, artificial harbors, tramways, and other complex facilities will require speed of construction and great effort.

The final heading under which we shall consider logistic effects of the new weapons and developments in warfare is management and organization. I shall not further belabor the trend toward maximum use of cross, common, and joint support. The Congress is tending to that by law, and the Executive Departments by regulation. Accordingly, and fortunately for all us

taxpayers, those trends are built in and it will be easy for us to get used to them. None of us enjoyed the spectacle, for example, of watching similar facilities, in competition for labor, go up within a few miles of each other. Increased common, cross, and joint support will be a trend; the only question concerns the extent. The extent that is optimum is a matter for determination by continued study at departmental levels. Maximum coordination must be exercised by each unified command headquarters overseas.

In the past logistic organizations, such as communication zones and base areas, just grew up like Topsy and were loosely knit. They grew up, then died away. They were never permanent. As they began to get used to working together, they would be closed out. The establishment of permanent overseas bases and communications zones in peacetime reflects a new trend. In addition, Army logistic commands and Navy functional components are being trained here in the United States. These are flexible units with trained nuclei that can be expanded or contracted as needed and that can be shifted, as combat units are now.

I have discussed the trends toward economy, speed, dependability of the flow of supplies, and development of indigenous resources. We very briefly and separately visualized how the probable nature of a future large-scale war would affect supply, medical service, transportation, construction, and management. We have recognized that more supplies, service, transportation—more of everything—will be needed.

The new weapons and equipment generally will be complicated, difficult to produce, and costly. The trend of the unit cost of killing an enemy is interesting and bears out the contention that more of everything will be required. Taking the cost of a war and dividing by the number of people killed, we find Caesar got away at 75¢ a man. Napoleon almost bankrupted France because, in his day, the price got up to $3,000 to kill a man. In World War I it cost us $21,000 per dead enemy, and in World War II it jumped to at least $200,000. That is the trend—75¢ to $200,000 [14]!

That is the heart of the problem. More of everything is required and everything is more expensive and harder to get. The new weapons will be able to damage our production facilities in the ZI and force our dispersion in the Com Z. More supply and service are required to support combat, yet are made more difficult by the necessity for dispersal and increased defense. Large depots, ports, and airfields in the Com Z would invite disaster. More transportation is required, but the tonnage carriers—sea transport and railroads—will be in great jeopardy. There is a dilemma. Due to the new weapons more and more is needed, yet less will be available. That, logistically, is intolerable. We have made revolutionary improvement in our weapons, leaving a gap between our offensive capability and the defense. But there is a greater gap capable of developing between combat operations and their proper logistic support.

Consequently, we must come to the aid of logistics with frugality, speed, accuracy, improved organization, and perhaps with the development of new sources of materials. As an interim measure we will have to be as alert to what we can get along without as we always have been to what we need.

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STATISTICAL METHODS FOR DETERMINING REQUIREMENTS OF DENTAL MATERIALS

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The prediction of future requirements is an important factor in the effective and economical operation of many organizations. The analysis of factors determining the optimum balance between reasonable cost and reasonable availability came to be an important aspect of operations research during World War II. The necessity for cautious use of certain critical supplies (e.g., rubber, tin) not readily available in our continent put an added burden on requirements planning. In peacetime, then, it becomes important that we study our past experience, devise methods for facilitating rapid adjustment to wartime needs, stockpile key items wherein that is practicable, and in general, be ready for immediate action.

The application of these principles to dental logistics is no less important to the successful performance of the dental mission. The determination of the future requirements of dental materiel is most easily accomplished by using past issues as a basis for predictions. If operating conditions remain the same, undoubtedly this method is quite adequate. When one is certain that conditions will not be the same in a future year, an adjustment factor must be applied. This factor might be established by developing a direct proportion such as the following: If 100 items were adequate for 1,000 men in the past year, then 200 will be required for the next year if there is to be an increase of 100 per cent in personnel. This is an adequate method, providing usage actually increases in direct proportion to personnel.

An examination of the specific conditions which pertain to usage of dental material may lead to some conclusions as to where it would be most profitable to begin a search for the correction factor to apply. For example, if the Navy increased from 500,000 total personnel to 1,000,000, but the dental officer strength remained the same, would one expect an increase in usage of dental material of 100 per cent or zero—or somewhere in between? Also one might inquire as to the effect of increase in number of dental officers on usage of dental material, when the ratio between dental officer strength and total personnel was constant. One must assume that a single dentist can accomplish more if he has good assistance, improved working conditions, or if he works longer hours. In spite of these recognized variables, it appears to be quite certain that the number of dental officers employed would offer a more reliable index for establishing requirements than would the total number of personnel. In a previous study [1] in which data from four war years (1942-45) were used, there was evidence that a positive relationship existed between dental officer strength and issue of materials, though the relationship was not directly proportional. It seems to be worthwhile to make a more detailed study of

these values and to extend the period to be analyzed so as to include both wartime and peacetime usage. Only typical items need to be studied.[1]

There are numerous items of supply and equipment which are issued in comparable quantities, even though their physical or chemical characteristics are not the same. It is obvious, for example, that dental operating chairs and dental operating units will be required in about the same numbers. Where issues for such items are similar, only one of them is studied. Figure 1 shows two pairs of such items, the similarity of which may be easily noted by simple plotting of issue quantities at regular intervals.

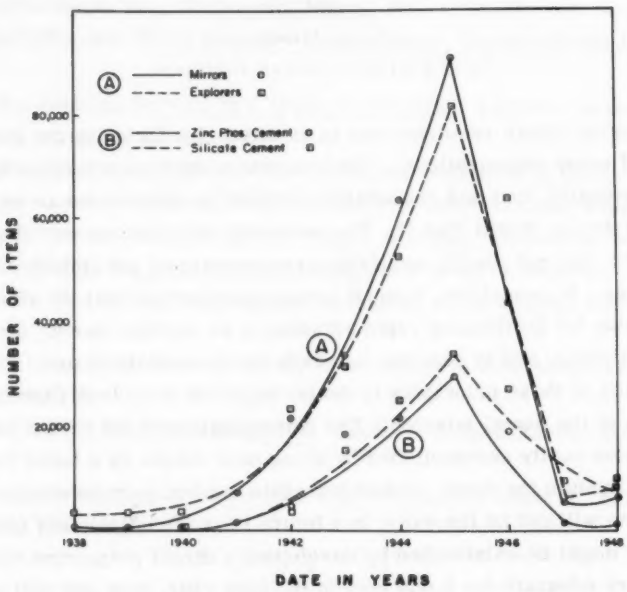

Figure 1

In order to establish the relationship between the number of dental officers on duty with the quantities issued of a particular supply item, a plot of these factors may be made on rectangular coordinates. Figure 2 shows a plot of one typical supply item—gauze napkins. Here the quantities of gauze napkins issued are plotted on the vertical axis and the number of dental officers on the horizontal axis, because we wish to express issues as a function of the number of officers on duty. Since the plotted points do not fall on a straight line, we know that the relation between the variables is not of the first degree. That is, if with each increase of one dental officer there was an increase of the same number of napkins, the plotted points would

[1]There are about 800 items of supply and equipment which are specifically used in dentistry and perhaps 200 others that have common usage in dentistry as well as in other professions (such as certain x-ray supplies, optical pliers, and medical items). In order to cut down the number of individual items that would be carried through a series of studies, things identical except for size, such as dental burs, are considered together when similarity of issues warrants it. The same applies to instruments that are supplied in right and left pairs, such as hatchets and excavators, or sets, as is the case with apical elevators.

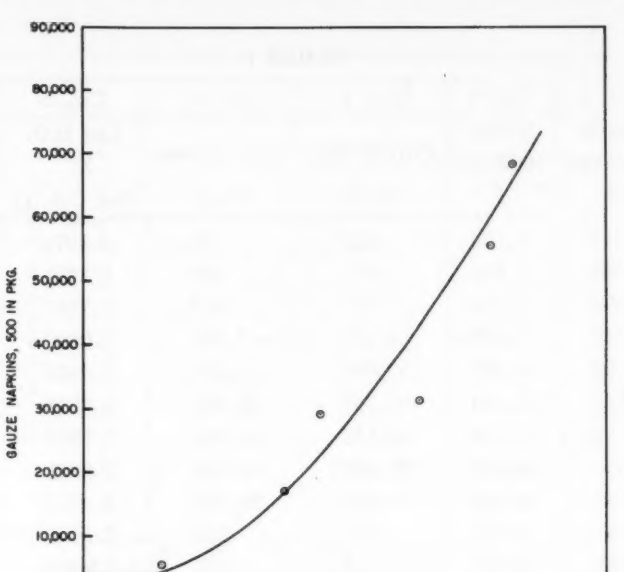

Figure 2

have fallen along a straight line. The fact that a smooth curve can be drawn so as to pass through or very near the plotted points indicates that some relation may exist between these two variables, which may be represented mathematically by the equation of the curve. The data from which this plot is made are found in Columns 1 and 2 of Table I.[2]

It is desirable to secure a simple expression for the relation between the required number of units of dental supplies and the number of dental officers when the latter number is changing rapidly, as in wartime. The shape of the curve formed in Figure 2 is parabolic and the relation appears to be a power function, of the form

(1) $$Y = a X^b$$

Y = number gauze napkins
X = number officers
a and b are constants.

We would like to verify the accuracy of this equation as a representation of our data. The usual way of doing this is to plot the values of X and Y on log-log paper and observe whether

[2]The basic data consists of the issue quantities of each item by fiscal years beginning July 1, 1937 (F.Y. 1938) and ending June 30, 1948 (F.Y. 1948). The records of these issues were made available by the Naval Medical Material Office, Brooklyn, N. Y., where a composite record is kept of issues from all supply depots and storehouses. (Prior to 1944 all new material was taken up at the main depot and was considered to be "issued" when it left that depot even though it went to another depot or storehouse). In some instances among a few items studied, unit packaging differed during the included years, therefore conversions were made to a single unit. During the eleven years included in the study, several different bookkeeping systems were adopted and discarded, but there was contiguity in all selected items. The source of information for officer strength came from the Dental Division of BuMed.

TABLE I

Years	Col. 1 Dental Officers X	Col. 2 Gauze Napkins Y	Col. 3 Col. 2-3380 (Y-c)	Col. 4 Col. 2-2500 (Y-c)	Col. 5 Log D.O. X' (log Col. 1)	Col. 6 Log (Y-2500) Y' (log Col. 4)
1938	250	2,572	-808	72	2.3979	1.8573
1939	268	2,806	-574	306	2.4281	2.4857
1940	523	2,771	-609	271	2.7185	2.4330
1941	1,213	5,530	2,150	3,030	3.0803	3.4814
1942	3,152	16,966	13,586	14,466	3.4986	4.1604
1943	5,268	31,143	27,763	28,643	3.7216	4.4570
1944	6,246	55,518	52,138	53,018	3.7956	4.7244
1945	6,580	68,246	64,866	65,746	3.8182	4.8179
1946	3,643	29,075	25,695	26,575	3.5615	4.4245
1947	881	2,633	-747	133	2.9450	2.1239
1948	811	3,374	-6	874	2.9090	2.9415

$\Sigma X' = 34.8743, \quad \Sigma Y' = 37.9070$

$\Sigma X'^2 = 113.4194,^* \ \Sigma X'Y' = 125.8670^*$

*Computed directly on machine from Cols. 5 and 6.

a straight line is approximated by the points. If our equation (1) does describe our data, this method will produce a straight line because by taking the logarithms of both numbers of this equation we get

(2) $$\log Y = \log a + b \log X,$$

and if we let $X' = \log X$ and $Y' = \log Y$ this becomes $Y' = \log a + b X'$ which is an equation of the first degree in X' and Y'; therefore, a plot of (X', Y') or $(\log X, \log Y)$ must approximate a straight line.

In Figure 3, Curve (a) is obtained when the data used in Figure 2 are plotted on log-log paper and it will be observed that a straight line cannot be considered an adequate representation of the points.

In seeking for a correction of our first expression (1) which will produce an adequate representation of the relationship between our variables Y and X, the simplest modification takes the following form

(3) $$Y = a X^b + c$$

which by transposition of c becomes

(4) $$Y - c = a X^b$$

which reduces to

(5) $$\log (Y-c) = \log a + b \log X.$$

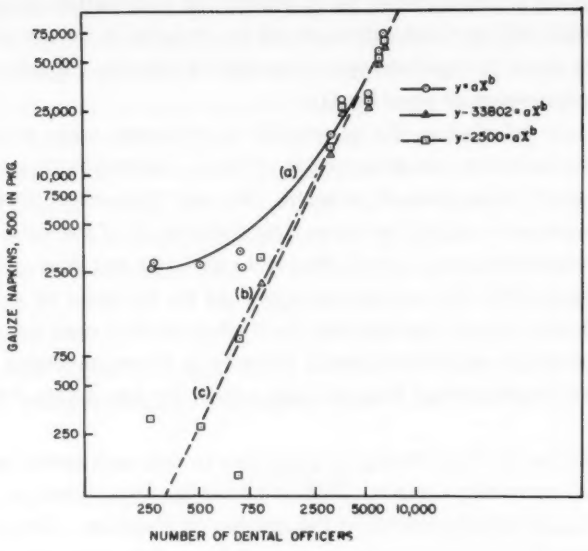

Figure 3

Here again we see that we have the general equation describing a straight line, the only modification being that a constant, c, is subtracted from Y. A plot of (Y-c) against X should give a straight line on log-log paper if our modified equation (3) adequately represents the data. First, of course, we must find the numerical value of c in order to carry out the procedure of plotting. The estimation of c can be done mathematically as follows: Choose two points on the curve and note the values of X and Y for each: $X_1 = 1000$, $Y_1 = 4550$; $X_2 = 7000$, $Y_2 = 74{,}500$. Choose a third point on the curve such that $X_3 = \sqrt{X_1 X_2}$ and note the Y_3 value for this selected point, as

$$X_3 = \sqrt{X_1 X_2} = \sqrt{1{,}000 \times 7{,}000} = \sqrt{7{,}000{,}000} = 2645.8$$

and reading the Y_3 value from the plotted curve

$$Y_3 = 12{,}500$$

c may be obtained (as given in [2]) from

(6)
$$c = \frac{Y_1 Y_2 - Y_3^2}{Y_1 + Y_2 - 2Y_3}$$

$$c = \frac{4550 \times 74{,}500 - 12{,}500^2}{4550 + 74{,}500 - (2 \times 12{,}500)} = 3380.66.$$

Having the value for c, our original data can be modified by subtracting c (calculated to be 3380) from the original issue quantities (Col. 2, Table I and the new values (Col. 3) are plotted against X (Figure 3, Curve b). Only positive values are used. The fact that this figure appears considerably more linear than the previous one is due in part to the elimination of Y values

below 3380. The dropping of these values is, in effect, the elimination of peace-time data, and may be an indication that only war-time data should be included if we are interested in the conditions under which there is rapid change in number of officers. If this is true, we may return to our simple expression of equation (1).

The mathematical calculation of c is actually an estimate, since selected points on our plotted curve are used. Actually c is an estimate of the Y- intercept obtained from the plotted line in Figure 2 (i.e., the Y value when X is zero). We may then select arbitrarily a value for c which appears to be adequate and at the same time retains all of the data. Thus we may select the largest round number for c which does not eliminate any data. From Figure 2 we note this would be around 2500. We may proceed now as we did when we selected 3380. Column 4 of Table I shows the values obtained for (Y-c) when 2500 is used as the c value, and a log-log plot of this against the number of dental officers is shown as Curve (c) in Figure 3. It shows considerable rectification over Figure 2 and except for two points (1939-47) is very good.

The equation (5) for the best fitting straight line is then calculated by finding the numerical values of the constants a and b. This requires the logarithms of the number of dental officers (Col. 5) and the logarithms of the number of (napkins - 2500) (Col. 6), which are given in Table I. The method of least squares is used for solution by substituting in the following equations and carrying out a simultaneous solution:

$$\Sigma y = \alpha n + b\Sigma x$$
$$\Sigma xy = \alpha\Sigma x + b\Sigma x^2,$$

where

$$y = \log(Y-2500)$$
$$x = \log X$$
$$\alpha = \log a$$

Then from Table I,

$$37.9070 = 11\alpha + b34.8743$$
$$125.8670 = 34.8743\alpha + b113.4194.$$

Multiplying each member of the first equation by the coefficient of α in the second equation (34.8743) and carrying out a similar procedure in the second equation (using 11), it becomes possible to subtract the equations and eliminate α. The value for b may be obtained by solving the resulting equation (b = 1.9925), and the value for α may be obtained by substituting this b value in either equation and solving (α = -2.8709). The formula for calculating either Y or X is now complete:

$$\log (Y-2500) = 1.9925\log X - 2.8709.$$

The method described above provides a formula for determining the number of gauze napkins required if the number of dental officers on duty is known. Rather than actually using the formula, graphic extrapolation from a plot of the data from previous years and knowledge

of the officer strength for the year in question may be used to determine the same information. That is, the known officer strength can be plotted on the curve, and the pertinent supply information read on the vertical coordinate. The maintenance of a set of curves on each item of supply would not require each one to be carried through the procedures described above. Our study indicates that about half of the dental items supplied to the Navy can be fitted to formula (1)—i.e., only two constants are involved, and a graph of the basic data on log-log paper provides a usable tool (Table II). Other items studied, with the exception of prosthetic materiel, were adequately rectified to a straight-line-plot by using a value c which was an estimate of the Y intercept, yet retained all of the data. For example, in the case of the Howard Apical Elevator, No. 1, (Table II, item 11) the log-log plotted curve of the basic data was not a straight line. Upon extension to the Y axis, the intercept value was about 20. When (Y-20) was plotted against the dental officer strength (X), a reasonably straight line could be plotted and no further calculation was performed. Perhaps a third of the items could be plotted in a similar fashion.

TABLE II

Selected Typical Items or Groups	Formula Fitted			Number of items represented
	$X = aX^b$	$Y - c = aX^b$	Double curve	
1. Silver alloy		X		1
2. Burs, mirrors, explorers	X			20
3. Bracket table covers		X		2
4. Cements, cellulose strips, wedges	X			10
5. Amalgam pluggers (Black No. 1 or Sweeney No. 7)		X		8
6. Wedelstaedt chisel, 41-42	X			18
7. Gregg plastic instruments, 2-3	X			8
8. Gauze napkins		X		1
9. Gauze stripping, unmedicated		X		6
10. Forceps, 150a-151a	X			10
11. Elevator, Howard, N. 1		X		20
12. Impression compound, red			X	4
13. Dental stone			X	5
14. Articulator, crown and bridge		Irregular		2
15. Laboratory chisel, left		Irregular		

Special consideration should be given to prosthetic items. The number of dental officers engaged in prosthetic dentistry suddenly changed in 1941, when the dental standards for naval enlistment were altered so as to permit practically anyone without gross oral infection to enter the Service. Almost overnight it became necessary to establish large prosthetic clinics. This sudden change in circumstances naturally was reflected in the quantity of prosthetic material issued. Therefore, in the case of prosthetic items, as exemplified by Dental Stone (Figure 4), the relationship is best represented by two curves: one for peacetime years and one for the five war years. The principle of using two curves for data collected under dissimilar circumstances is statistically sound ([2], p. 161), and in this case seems to be warranted. If data could be made available as to the number of dental officers engaged in

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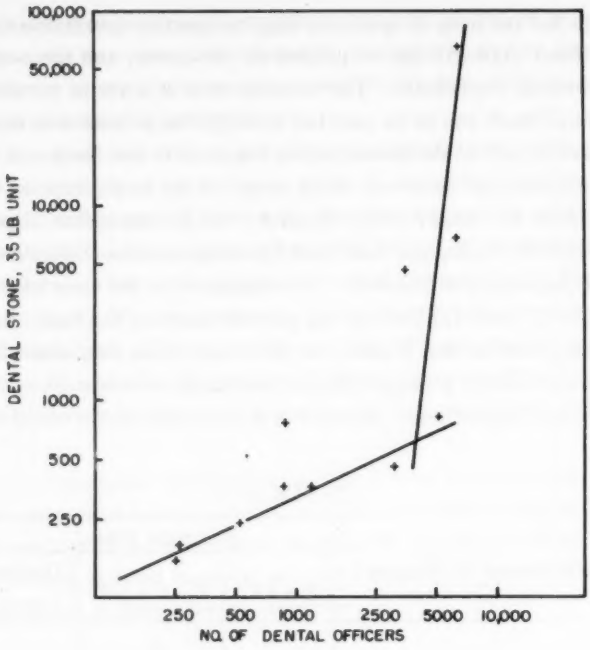

Figure 4

prosthetic dentistry, this would be a better figure to use for the X value than the total number of dental officers. Since prosthetic dentistry has not reverted to the status it had prior to World War II, it may be expected that not so drastic a change would be observed with the advent of another war.

With each new procurement period, whether a formula or curve is used in predicting the needs, verification of the adequacy of the method will be observed by seeing how accurate the prediction was in the previous period. A constant correction of the curve (or formula) is thus instituted. This is much safer than using a single previous year's needs as an index of the amount to purchase the following year. If there is a new period of military expansion, experience gained from the last war will be added to experience gained during the expansion.

In the last analysis, of course, no mechanical method can be adopted which will satisfactorily determine requirements without the thoughtful application by a person who is thoroughly familiar with the physical characteristics of the items, their professional use, and the general problems of supply. In the past, substitutions have been made of available items for unobtainable items whenever it was professionally feasible. For example, dental burs were in short stock during World War II, and it was necessary to meet the needs for them by issuing approximate sizes, resharpened burs, and by making available diamond drills and small dental stones [3]. Such procedures throw a discrepancy into the record of issues and must be taken into account in using these records as a factor in establishing requirements. A similar situation results when new items are made available that duplicate the use for which other things are already supplied. With watchfulness and care, troublesome items (perhaps 10 per cent) can be isolated and resolved by special methods. For the preponderate majority of items, we believe an application of the method described herein will markedly facilitate determining requirements.

SUMMARY

Selected items of dental supply are analyzed as to their usage in the U. S. Navy in order to develop a relationship between issues and the number of dental officers providing dental service. A period of eleven years' (1937-1948) operations are involved in the study, which includes a drastic increase in dental personnel and patients to be treated. Using the method of least squares, it seems possible to develop formulas and graphs which will provide valuable assistance in determining requirements for dental material during periods when rapid personnel changes are occurring. The need for caution is pointed out, both as to the blind use of formulas and as to the need for routine correction of formulas as new data becomes available.

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LOGISTICS—THE WORD

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The word, LOGISTICS, is a U. S. Navy contribution to the American military language which is now the military language of the Free World. It was introduced by none other than Rear Admiral Stephen B. Luce, founder of the Naval War College.[1] Luce [1] introduced the word in 1885 in connection with announcing the prospective course for the College in an article in The United Service, a scholarly journal of the time devoted to the interests of both the U. S. Army and Navy. The first Army use of this term was by General William T. Sherman [2] in an article in 1888.

Luce borrowed the word LOGISTICS for naval purposes from Jomini,[2] the French writer on war who had served with Napoleon. Jomini had divided the art of war into six distinct parts or categories: (1) Statesmanship; (2) Strategy; (3) Grand tactics or the art of the general; (4) Logistics; (5) Engineering of the attack and defense of fortifications; and (6) Minor tactics or the movement and training of formations. In his article Luce defines a naval application for each of these categories. His definition for LOGISTICS should be quoted:

> LOGISTICS, or the art of moving armies, finds its counterpart in the art of navigation, and in securing adequate supplies of provisions, ammunition and fuel for the fleet, when severed from its immediate base of supplies. This requires a knowledge not simply of the ordinary rules of navigation, which includes a knowledge of the laws of aberration of the compass in metal ships; but of the coal depots of the world, the means of obtaining supplies abroad, and the facilities afforded in different foreign stations for docking and repairing ships, and making duplicate parts of machinery. It requires also a knowledge of the different channels of an enemy's ocean commerce, with a view to sustaining a fleet on the products of that commerce. Information necessary to cover these points will be one of the duties of the naval staff, a corps it will be an object of the Naval War College to produce.

The subject of logistics received considerable attention in the period before World War I when a growing navy forced attention on coaling stations, maintenance facilities and service ships for maintaining and supplying the fleet overseas. Mahan [3] used the term

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[1] The Naval War College was established by General Order No. 325 of 6 October 1884. Luce was the first President, A. T. Mahan, the second. The first session, lasting one month, opened on 3 September 1885 with a class of eight.

[2] Logistics appears in the French language before Jomini's use of the word. It was derived from the French word, "logis," which means quarter, in the sense of place to live, and is the same as occurs in the English word, "Quartermaster." In the German use of the word, logistics, it was related to logic, being derived from the Greek word, "logos," which means mind. It was used to distinguish mathematical logic from ordinary logic. (This footnote is based on a communication received from Professor Oskar Morgenstern.)

LOGISTICS in a lecture at the Naval War College in 1888. Logistics became of paramount importance during the cruise of the fleet around the world from 1907 to 1909 and also with the establishment of naval stations at our newly acquired overseas possessions.

Many articles on the subject by naval officers appeared in service journals and in those for the general reading public. Admiral W. L. Rodgers, who served both on the faculty of the Army War College and as President of the Naval War College, was farsighted enough to include economics with strategy, tactics and logistics as the four elements of warfare [4]. Admiral T. A. Cowie (SC) produced a document in March 1917 on the subject of what now would be called Economic Potential for War. It was entitled "Logistic Data on Production and Industry in the United States." The data were arranged under the headings of: (1) Forests and Forestry; (2) Mining; (3) Farming and Agriculture; (4) Manufactures; (5) Internal Communications and Transportation; and (6) Occupations. It showed comparisons, probable requirements and suggested actions to be taken in event of war to provide material and personnel. In the same year Colonel G. C. Thorpe, USMC brought out his little classic, Pure Logistics, The Science of War Preparation. He also had an article in the U. S. Naval Institute Proceedings for 1917 on how a correct strategy, tactics and logistics can be integrated and determined by the Estimate of the Situation discipline.

It was during the period between World War I and II that logistics became a second class subject in the naval profession. The growth of our Navy had stopped and the stress was on the operations, usually in home waters, of the ships that we had, on tactics and ship handling, subjects that Jomini called "minor tactics." Most fleet problems of those days ended at about the time when logistics was coming into play and would begin to influence commanders' decisions. The Navy paid dearly in the early days of World War II for this attitude toward logistics, as Duncan Ballantine points out in his book, Naval Logistics in World War II.

The use of the term LOGISTICS has come into general use in the other services only recently. The excellent final report of the Army Service Forces, Logistics in World War II, did much to encourage this although acceptance is not yet universal as is indicated by the continuing discussion of Admiral Eccles' article, "Logistics—What Is It?"[3]

The Navy's early acceptance of this French word was due to Luce's advocacy of using military history to teach the science of naval warfare. This was not too favorably received by his naval contemporaries despite the fact that several years would elapse before Mahan would produce the first classic on naval warfare. Jomini's book, The Art of War, was a standard text in the U. S. Army in the second half of the nineteenth century. It had been translated by two Army officers at West Point in 1862. Clausewitz's work, On War, on the other hand, was translated into English for the first time in 1873, after the German successes against Austria and France. This book then became the gospel for army officers the world over. The French word, LOGISTICS, is not found in Clausewitz, hence its limited and only recent acceptance outside of the Navy.

Admiral Luce's achievements and his contribution to the efficiency of the modern U. S. Navy are many, but they are almost unknown for they were in the fields of education and organization where fame in the military profession is not easily gained. The question, "Logistics—What Is It?" was considered in 1885 by this remarkable man whose biographer says, "To such as he there can be no successor."[4]

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[3]See "Discussions," U. S. Naval Institute Proceedings, August 1954.

[4]Last line in Life and Letters of Stephen B. Luce, by Admiral Albert Gleaves, New York, 1925.

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* * *

DEVELOPING AN ENGINEERED PRODUCTIVITY STANDARD

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One of the most common problems of persons concerned with productivity analysis is that of developing a standard which will not only spotlight problem areas but actually be sensitive to virtually all the work being done by a group of people. Such a standard is classified as "engineered" by statisticians. It identifies itself by its complexity (more than one work unit) and resultant more costly development than the more common "statistical" standard. The statistical standard generally consists of one unit of measure representing the output of a group of people. Without such a standard, however, the true value of a productivity analysis cannot be obtained because generally a comparison of actual performance against the statistical standard results in merely spotlighting an area which appears to be over-staffed. The determination of the quantity value of the over-staffed condition can only be arrived at by a more detailed study of productivity. This means that for such a detailed analysis something in the form of an engineered standard must be available.

The detail method of approach in developing an engineered standard will vary depending on the specific problem at hand. For some types of studies it will be necessary to run time studies to get accurate estimates of the time required to produce a unit of work. In other cases it may be satisfactory to use rough estimates where precision is not required.

At the Naval Supply Center, Oakland, California, the problem has been under study for some time. We have developed what we believe to be a simplified approach to the problem.[1] So far, the plan has proven to be sufficiently workable to be considered an appropriate subject for a paper of this kind. The concept seems to be simple to understand and a workable one which could be modified and adapted to most any function or organizational component of an activity in either government or industry.

The first phase of the study was to analyze the organization, office layout, functional processing steps, and document flow. This information was put in chart form. The next phase was to determine which groups of people were doing similar work in order that a work unit selection could be made which would generate productivity standards for each person within the group. The survey showed that for each of the three main units in the branch under study each of the persons in a group were doing sufficiently similar work to warrant the development of productivity standards against which the individual performance can be evaluated.

A form was developed to collect manhours and work units during two sample periods (two weeks). From the statistics collected, averages were computed and discussed with the branch and unit supervisors. Necessary adjustments were made and agreed upon by the supervisor to eliminate radical variations which did not appear to represent normal conditions or

[1] U. S. Naval Supply Research & Development Facility, NSD, Bayonne, N. J., Progress Report (Project NT004006) of 31 December 1952, was used in developing this study.

performance. Following this test period tentative standards were established and another test made about one month later. Firm standards were then established.

The method of evaluating performance once the standards for the various functions were established was as follows:

FOR THE INDIVIDUAL

a. For a given survey period, volume of production and manhours expended were collected for each of the functions having standards. The number of manhours used to accomplish one work unit were then computed for each function. Total manhours of effort for the overall production covering all functions were accumulated.

b. From the standards, manhours which should have been used to accomplish one work unit were computed for each function.

c. Through the application of the standard manhour requirements to each production volume accomplished during the survey period, an aggregate of total standard manhours required for the work accomplished were computed.

d. A comparison was made of the actual aggregate manhours used for a given volume of work units with the standard aggregate manhours for the same volume of work. By dividing the standard by the actual an index of performance expressed percentage-wise resulted.

e. By subtracting the standard from the actual any excess hours were revealed. Conversely, if actual was less than standard, improved performance was revealed.

It is pointed out that each person involved in the accomplishment of several different types of work will have an overall standard which will vary, depending on the combination of work (and standards) which the individual is concerned with. Expressing this concept with symbols, we have the following formulas:

1. Requisition Processing

$$H_1^s = b_{11} X_1 + b_{12} X_2 + b_{13} X_3 + b_{14} X_4$$

where H_1^s = standard man hours for processing a given number of different types of requisitions (the first function).

b_{1i} = standard amount of hours to process one specific type (the i-th) of requisition, in the first function.

X_i = number of a specific type of documents (the i-th).

X_1 = master requisition
X_2 = emergency requisition
X_3 = suspense copy of requisition
X_4 = scheduling a requisition

$$H_1^s = \sum_{i=1}^{4} b_{1i} X_i$$

2. Completion of folders

$$H_2^s = b_{25} X_5$$

where b_{25} = standard man hours to complete one folder
X_5 = number of folders completed

3. Follow-up

$$H_3^s = b_{36} X_6$$

where b_{36} = standard man hours to do one line item of follow-up.
X_6 = number of follow-ups (L/I) accomplished.

Having computed the standard man hours to accomplish a given volume of work units for each of the functions discussed above, we can then put the three values together and arrive at an aggregate volume representing the standard man hours that should have been used to accomplish any given combination or volume of work units. This can be expressed as follows:

$$H^S = \sum_i H_i^S$$

By substituting the actual man hours "a" used to accomplish the different types of work units for the standard hours "s" in the above formulas, we can then compute the total actual hours used to complete the total aggregate of work units. We should then have two sums:

$$H^a = \sum_i H_i^a \text{ and } H^S = \sum_i H_i^S$$

and if we consider one individual k we can have

$$\frac{H^{sk}}{H^{ak}} = \text{Index of performance } (P_k).$$

FOR THE GROUP

The performance of a group of people was evaluated, using the same concept discussed above. Having an actual and a standard man hour figure for each person, it was then possible to summate the actual hours for all persons in the group and likewise add all the standards for the same group. We then have an aggregate standard and an aggregate actual for a given group of people which can be expressed as follows:

Actual hours for group

$$T^a = \sum_k H^{ak}$$

where T^a = Aggregate actual hours for a group.
H^{ak} = Actual hours for person k.

Standard hours for group

$$T^S = \sum_k H^{sk}$$

where T^S = aggregate standard hours for a group.
H^{sk} = standard hours for a person.

We then have

$$\frac{T^S}{T^a} = \text{Index of performance } (P_G) \text{ for group.}$$

CONCLUSIONS

It has been the purpose of this paper to explain one concept for the solution of the perennial problem faced by the production analyst that "the statistics show we are bad but ..... how bad?" Limitations in funds to support the collection of detailed statistics usually result in the policy of using the broad "statistical" approach until it appears that potential savings will offset the additional cost to make a more detailed or "engineered" analysis of productivity by person. The proper application of the formulas should result in a revealing of those areas of indirect labor type work which are not being performed up to standard and which so often are buried in the overall statistical rates. Periodic operational research of this nature should pay high dividends to management, particularly in "paper mill" type functions where it is difficult to find a single work count indicator.

* * *

THE QUARTERMASTER CORPS: ORGANIZATION, SUPPLY, AND SERVICES, VOLUME I — A BOOK REVIEW

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U. S. Navy (Retired)

This article is a review of the book, The Quartermaster Corps: Organization, Supply and Services, Volume 1, by Erna Risch, Office of the Chief of Military History, Department of the Army, 1953, 418 pp. This book can be purchased from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

This is the seventeenth volume in the monumental series U. S. ARMY IN WORLD WAR II being prepared by the Chief of Military History of the Army. It is the first of four books in the Technical Services sub-series that will deal with the manifold and vital duties of the Quartermaster Corps.

This book and the others in the series illustrate an important difference between the fundamental thinking of army officers and naval officers. The comparison between the way the Army and the Navy have approached the history of World War II is therefore illuminating.

The Navy history consists of an integrated series of fourteen volumes on naval operations, individual books on submarine, aviation, marine and amphibious operations, three almost casually related books on mobilization, national naval logistics, and mobile logistic support, a few Bureau and special histories. The Army is producing a huge, fully integrated series in which nine volumes are devoted to overall command and organization, thirty volumes to various logistics matters, and thirty-two to the various theaters of operations. The remaining seventeen volumes deal with special studies and civil affairs. In addition the University of Chicago is publishing seven volumes on the Army Air Forces.

The Navy books are in a variety of format, are relatively brief, and in many cases highly personal; the Army series are in a standard format, are massive and somewhat impersonal.

This volume contains chapters on organization, research, clothing, personal and organizational equipment, subsistence requirements, procurement, production, storage, and stock control. It is well indexed and contains a useful glossary.

The book will be of particular interest to officers of the Supply Corps to whose activities the Quartermasters offer a close parallel. However, Chapter I on organization and Chapters VII and VIII on procurement and production offer many points of interest to all persons concerned with industrial and economic mobilization and other high level logistic policy and planning.

The section on organization brings out another instance of the almost unending series of reorganizations that take place in our national efforts. Each always is hailed as "The Right Answer" and yet when tested by war almost every "Right Answer" turns out to be inadequate.

One wonders if this happens because we are unable to visualize in advance the problems of war, or whether it is because we are unwilling to face the implications of so doing.

In the story of procurement there are good discussions of the change over from centralization to decentralization of the development of consolidated purchase, negotiated contracts and other measures used to facilitate procurement.

Considerable emphasis is placed on the hampering effect of restrictive legislation, both national and state, that always adds so much to the time and expense of doing business. For example on page 258 we find:

> The necessity of obtaining special clearances—frequently in each individual case—from the Federal Prison Industries, Inc., and the Committee on Purchases of Blind-Made Products before certain of their items could be purchased from commercial sources hampered expeditious procurement. Even when clearances were issued the administrative procedures were time consuming. Each clearance had to be published in a circular letter, and all posts, camps, and stations had to keep their files up-to-date and cite a clearance on every purchase order placed outside the schedules.

The effect on procurement caused by placing contracts in areas where there are labor shortages is discussed and attention is invited to the neglect of this factor in prewar planning.

Pages 269 to 271 contain a valuable discussion on the development of cost studies, cost analysis and forward pricing as a means of preventing excess profits and improving efficiency.

The chapter on production contains many passages which recall comparable experience of the Navy.

In the light of the many pleas in the Congress for the better utilization of small business in defense production it is interesting to note how inexperienced business firms repeatedly over-estimated their ability to produce.

> Processing difficulties and managerial problems further served to delay production on Quartermaster contracts. Not all Quartermaster suppliers were financially sound, reasonably alert to improvements in manufacturing methods, and gifted with high standards of business ethics.
>
> Only if suppliers' capacity, equipment, and ability were analyzed prior to the placement of orders, and only if adequately equipped, qualified, and diligent suppliers were allowed to undertake performance on contracts, could such delays be avoided. If marginal producers had to be brought into production, it was imperative to give them all possible help with their problems.

The discussion of Priorities and Allocations on pages 290 to 296 is of the greatest importance. The following comments indicate some of the problems:

> To guide and direct the flow of critical materials into military and essential civilian channels and to attain maximum production for war, the nation's defense program relied upon a patchwork of controls developed during the emergency and war periods.
>
> * * * * * *
>
> From the beginning more and more Army and Navy projects were placed in the top priority category, inflating it to the point where in order to relieve the congestion in the A-1 band, a graduated priority status had to be established.
>
> * * * * *

In the case of items not commercially procurable, the situation was soon acute. In these fields the Corps competed directly with other supply arms and services and with bureaus of the Navy, which were authorized to assign a much higher rating for similar equipment. Not infrequently, supplies on order by the Corps were "lifted" by other services by the simple method of placing higher ratings on them.

* * * * * *

Because this directive could be variously interpreted, critical materials continued to be siphoned off the projects with higher ratings.

* * * * * *

Instead of concentrating complete production programs in one level of priority, the ANMB proposed a percentage assignment whereby at least partial top priority would be provided to all programs to permit balanced production. Under this arrangement, the priority assigned to each production program would be spread over three or four grades, and no individual program would receive top priority in its entirety.

* * * * * *

Experience in the administration of the defense effort had early indicated the need for other types of controls to supplement the priorities system. Of these the most important were allocations which would gear the entire defense program to the available supply of critical materials. Only by allocating these materials to the end use could a balanced production program be realized and competition between the services to complete their requirements be eliminated.

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To one who has followed postwar education closely it is gratifying to note how many of the lessons of this book are reflected in the lectures of the Industrial College of the Armed Forces and comparable service schools.

An amusing instance of the unexpected uses to which carefully developed materials were sometimes put is the report on page 184 from the 2nd Infantry Division that the powdered lemon juice supplied with K rations was excellent for scrubbing and bleaching wooden floors.

The whole problem of research is given careful treatment. In this the importance of peacetime technical research is stressed and it is also pointed out that the value of research is greatly limited if it is not followed by adequate field tests. The Army experience with conflicting comments from the field commands and the reluctance of these commands to welcome research personnel brings to mind somewhat similar situations in our fleets. Of course, when this occurs the research forces have no other recourse than to simulate field and fleet conditions to the best of their ability and hope for the best.

In general this book is an excellent beginning on the history of the Quartermaster Corps. It is hoped that before the last of this sub-series is published, a chart comparison will be made between the variety, volume and value of Quartermaster supplies of World War II and other Army and Navy supply systems. Such a comparison will be of great value to the student of military affairs in that it will help him to make a better evaluation of the relative size and complexity of these various logistical operations.

Such evaluation is an essential beginning (but only a beginning) to the understanding of the various proposals and changes in our national defense organization and laws.

* * *

ON MULTI-COMPONENT ATTRITION GAMES[1]

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A model is described for military games consisting of a large number of identical successive engagements, without resupply, with a player being defeated when his supply of any resource is exhausted. A method is given for determining which player should win.

1. THE PROBLEM

We shall be interested in a zero-sum game between two players, I and II. The players begin with specified supplies of certain commodities and play a series of identical engagements, each of which results in diminishing the supply of some of the commodities. The game ends when the supply of any commodity of either player is exhausted, and the player whose supply of some commodity has been reduced to zero loses. If both players run out of something simultaneously, we shall arbitrarily award victory to Player I. Our problem will be, for a specified type of engagement, to determine those large initial supplies for which, with best play on both sides, I is practically certain to win.

More precisely, let I's(II's) resources consist of a(b) commodities, so that the initial supplies are specified by a vector $z = (z_1, \ldots, z_c)$, $z_i > 0$, $c = a + b$, with $z_1, \ldots, z_a$ $(z_{a+1}, \ldots, z_c)$ denoting I's(II's) initial resources. The engagement is specified by an $r \times s$ matrix $Z = \| z(u,v) \|$, $1 \le u \le r$, $1 \le v \le s$, and is played by I(II) choosing an integer u(v), each choice made without knowledge of the choice of the other player. The vector $z(u,v)$ thus jointly determined specifies the diminution in supplies resulting from the engagement. We shall suppose that

$$\text{(a)}\quad z_i(u,v) \ge 0, \quad \sum_{i=1}^{c} z_i(u,v) > 0 \text{ for all } u,v\,,$$

$$\text{(b)}\quad \sum_{v=1}^{s} \sum_{i=1}^{a} z_i(u,v) > 0 \text{ for all } u,$$

$$\text{(c)}\quad \sum_{u=1}^{r} \sum_{i=a+1}^{s} z_i(u,v) > 0 \text{ for all } v.$$

Requirement (a) excludes resupply and guarantees diminution of at least one commodity in each engagement, so that the game is certain to terminate. If (b) fails, say for $u = u_0$, then I can, by choosing $u = u_0$ in each engagement, maintain all his commodities at their initial levels; (c) excludes the analogous trivial case favoring II.

[1]This paper was written under Office of Naval Research contract number Nonr-1197(00).

It will be convenient to specify strategies for an infinite sequence of engagements. A strategy for I is then a sequence $f = \{f_n\}$, $n = 1,2,\ldots$, where f_n is defined on the set of all $n-1$ tuples $(z(1),\ldots,z(n-1))$, where each $z(k)$ is one of the elements $z(u,v)$ of the matrix Z, and f_n has values in the set P of all vectors $p = (p_1,\ldots,p_r)$, $p_i \geq 0$, $\sum_1^r p_i = 1$ (f_1 is simply a point in P). The interpretation is that, if the first $n-1$ engagements resulted in vectors $z(1),\ldots,z(n-1)$, then on the n-th engagement I will choose $u = 1,\ldots,r$ with probabilities $p_1,\ldots,p_r$, where $p = (p_1,\ldots,p_r) = f_n(z(1),\ldots,z(n-1))$. A strategy g for II is defined similarly, replacing P by the set Q of vectors $q = (q_1,\ldots,q_s)$, $q_j \geq 0$, $\sum_1^s q_j = 1$. Associated with each pair f,g of strategies is a sequence of random variables $z(1), z(2),\ldots$, the vectors resulting from the successive engagements when strategies f, g are used. Let m be the smallest integer for which some coordinate of $z - \sum_1^m z(i)$ is not positive, and let M(f, g) be the probability that one of the last $c-a$ coordinates of $z - \sum_1^m z(i)$ is not positive. We are interested, then, in the game, determined by Z, a, z, whose strategies are the sequences f, g defined above, and whose payoff to Player I is M(f, g). Denote the value of this game by v(Z, a, z). Our problem will be to determine, for fixed Z, a, the set of all z for which $v(Z, a, tz) \to 1$ as $t \to \infty$.

We may always, by a new choice of units, reduce the problem to one in which the initial supply of every commodity is unity. This amounts to replacing Z by Z', where $z'_i(u, v) = z_i(u, v)/z_i$, and replacing z by $\delta = (1, 1,\ldots, 1)$. Henceforth the reduced problem will be considered, and the primes will be dropped from the z's. In this form our problem is that of determining, for a fixed a, c, the set of all matrices Z for which $v(Z, a, t\delta) \to 1$ as $t \to \infty$.

2. THE RESULT

We shall state our main result in two equivalent forms.

THEOREM 1A: If there is a matrix $D = \| d_{ij} \|$, $1 \leq i \leq a$, $a+1 \leq j \leq c$,

$d_{ij} \geq 0$, $\sum_{j=a+1}^{c} d_{ij} = 1$ for each i, such that for every $\lambda = (\lambda_1,\ldots,\lambda_a)$,

$\lambda_i \geq 0$, $\sum_1^a \lambda_i = 1$, the value of the game with matrix $H = \| h(u, v) \|$,

$1 \leq u \leq r$, $1 \leq v \leq s$ is positive, where

$$h(u, v) = \sum_{i=1}^{a} \lambda_i \left(\sum_{j=a+1}^{c} d_{ij} z_j(u, v) - z_i(u, v) \right),$$

then $v(Z, a, t\delta) \to 1$ as $t \to \infty$.

THEOREM 1B: For any $q = (q_1,\ldots, q_s)$, $q_v \geq 0$, $\sum_v q_v = 1$, let T(q) denote the convex hull of the r points $\sum_v q_v z(u, v)$, $u = 1,\ldots,r$. If the set U of all points $z = (z_1,\ldots, z_c)$ for which $\max(z_1,\ldots, z_a) < \max(z_{a+1},\ldots, z_c)$ has a closed convex subset which intersects every set T(q), then $v(Z, a, t\delta) \to 1$ as $t \to \infty$.

THEOREM 2: The hypotheses of Theorems 1A and 1B are equivalent.

For $a = 1$ or $c-1$, the hypotheses of Theorem 1 are almost necessary:

THEOREM 3: For $a = 1$ or $c-1$, unless the set $\{\max(z_1, \ldots, z_a) \leq \max(z_{a+1}, \ldots, z_c)\} = \overline{U}$ has a closed convex set which intersects every $T(q)$ or, equivalently, unless there is a matrix D such that for every λ the value of the game with matrix H is non-negative, we have $v(Z, a, t\delta) \to 0$ as $t \to \infty$.

Theorem 1 is easily obtained from the following result, proved in [1]:

THEOREM 4: Let Z be any $r \times s$ matrix whose elements are points in c-space, let F, G be the sets of strategies f, g for I, II as defined in section 1, let $T(q)$ denote the convex set defined in Theorem 1B, and let S be any closed convex set. If S intersects every $T(q)$, there is a strategy f^* for I such that for any $\epsilon > 0$ there is an integer N such that for all g, Prob $\{\delta(\overline{z}_n, S) < \epsilon \text{ for all } n \geq N\} > 1 - \epsilon$, where $z(1), z(2), \ldots$ is the sequence of random variables determined by f^*, g, $\overline{z}_n = (z(1) + \ldots + z(n))/n$ and $\delta(\overline{z}_n, S)$ denotes the distance from the point $\overline{z}_n$ to the set S; that is, there is a strategy f^* for which $\delta(\overline{z}_n, S) \to 0$ with probability 1 as $n \to \infty$ uniformly in g. With the strategy g^*: $\{g_n = q\}$, $\delta(\overline{z}_n, T(q)) \to 0$ with probability 1 as $n \to \infty$ uniformly in f, so that unless S intersects every $T(q)$ there is no f^* for which $\delta(\overline{z}_n, S) \to 1$ with probability 1 for all g.

Roughly speaking, I can force the average payoff in a large number of plays to approach S if and only if S intersects every $T(q)$.

The application of Theorem 4 to Theorem 1B is immediate. For if S is a closed convex subset of U which intersects every $T(q)$, Theorem 4 yields a strategy f^* such that

$$\text{Prob}\{\overline{z}_n \in U \text{ for } n \geq N\} \to 1$$

as $N \to \infty$ uniformly in g. But for sufficiently large t, the game with initial resources $t\delta$ is certain to require at least N plays, so that

$$v(Z, a, t\delta) \geq \text{Prob}\{\overline{z}_n \in U \text{ for } n \geq N\}.$$

Consequently $v(Z, a, t\delta) \to 1$ as $t \to \infty$.

We turn to the proof of Theorem 2. Suppose the hypotheses of Theorem 1A are satisfied, and let S be the set of all $z = (z_1, \ldots, z_c)$ for which $z_i < \sum_{j=a+1}^{c} d_{ij} z_j$, $i = 1, \ldots, a$, so that S is an open convex subset of U. For any closed bounded convex T, S intersects T if and only if for every non-zero vector $\lambda = (\lambda_1, \ldots, \lambda_c)$,

$$\phi(\lambda) = \sup_{s \in S} (\lambda, s) > \psi(\lambda, T) = \min_{t \in T} (\lambda, t).$$

Since $s \in S$, $u \geq 0$ imply $us \in S$, $\phi(\lambda)$ assumes only the values $0, +\infty$. If $\phi(\lambda) = 0$, i.e. $(\lambda, s) \leq 0$ for $s \in S$, we must have (letting $z_i \to -\infty$, $i \leq a$), $\lambda_i \geq 0$ for $i \leq a$, and $(\lambda, z) = 0$ whenever $z_i = \sum_j d_{ij} z_j$, $i = 1, \ldots, a$, i.e.

$$\sum_{j=a+1}^{c} z_j(\lambda_j + \sum_{i=1}^{a} \lambda_i d_{ij}) = 0 \text{ for all } z_{a+1}, \ldots, z_c,$$

so that

$$\lambda_j = -\sum_{i=1}^{a} \lambda_i d_{ij}, \quad j = a+1, \ldots, c.$$

Thus S intersects T if and only if for every $\lambda = (\lambda_1, \ldots, \lambda_a)$, $\lambda_i \geq 0$, $\Sigma\lambda_i = 1$,

$$\min_{t \in T} \sum_{i=1}^{a} \lambda_i [t_i - \sum_{j=a+1}^{c} d_{ij} t_j] < 0.$$

The condition that S intersects every T(q) is that for every $\lambda_i \geq 0$, $\sum_1^a \lambda_i = 1$,

$$\min_q \max_u \sum_{v=1}^{s} q_v \left(\sum_{i=1}^{a} \lambda_i \left(\sum_{j=a+1}^{c} d_{ij} z_j(u,v) - z_i(u,v)\right)\right) > 0.$$

Since the left side is the value of the game with matrix H, the inequality is satisfied and S intersects every T(q). Then for some $\epsilon > 0$, the set $S(\epsilon)$ of all z for which $z_i \leq \sum_{j=a+1}^{c} d_{ij} z_j + \epsilon$ for $i = 1, \ldots, a$ intersects every T(q), and the hypothesis of Theorem 1B is satisfied.

Conversely, if the hypothesis of 1B is satisfied for a closed convex set S_1, we need only find a matrix D for which S_1 is a subset of the set S of points z for which $z_i < \sum_{j=a+1}^{c} d_{ij} z_j$, $i = 1, \ldots, a$, for we saw above that S intersecting every T(q) is equivalent to the hypothesis of 1A for the given D. Let S_2 consist of all points λs, $s \in S_1$, $\lambda > 0$ and let d_{ij}, $i = 1, \ldots, a$, $j = 1, \ldots, c$ be the coefficients of the hyperplane separating S_2 from $\{z_i \geq \max(z_{a+1}, \ldots, z_c)\}$: say

$$\sum_{j=1}^{c} d_{ij} z_j > 0 \quad \text{for } z \in S_2,$$

$$\leq 0 \quad \text{for } z_i \geq \max(z_{a+1}, \ldots, z_c).$$

From the last inequality, $d_{ij} = 0$ for $j \leq a$, $j \neq i$ (letting $z_j \to \pm\infty$), $d_{ij} \geq 0$ for $j > a$ (letting $z_j \to -\infty$), and $d_{ii} + \sum_{j=a+1}^{c} d_{ij} = 0$ (letting $z = \pm(1, 1, \ldots, 1)$). We may normalize by setting $d_{ii} = -1$, obtaining

$$z_i < \sum_{j=a+1}^{c} d_{ij} z_j \quad \text{for } z \in S_2.$$

Thus we have found a D as required, and the proof is complete.

We remark that the last steps in the above proof are essentially the proof that the maximal convex subsets of $\max(z_1, \ldots, z_a) \leq \max(z_{a+1}, \ldots, z_c)$ are the sets

$$\left\{ z_i \le \sum_{j=a+1}^{c} d_{ij} z_j,\ i = 1, \ldots, a \right\} \text{ for } d_{ij} \ge 0,\ \sum_{j=a+1}^{c} d_{ij} = 1.$$

To prove Theorem 3, note that Theorem 1B, applied to Player II, implies $v(Z, a, t\delta) \to 0$ as $t \to \infty$ if there is a closed convex set disjoint from $\bar{U}$ which intersects every set $R(p)$, $p = (p_1, \ldots, p_r)$ $p_u \ge 0$, $\sum_1^r p_u = 1$, where $R(p)$ is the convex hull of the s points $\sum_u p_u z(u, v)$, $v = 1, \ldots, s$. Now every $T(q)$ is a closed convex set which intersects every $R(p)$, since $\sum_{u,v} p_u q_v z(u, v)$ is in both. For $a = 1$, $\bar{U}$ is itself convex, so that the hypothesis of Theorem 3 asserts the existence of a $T(q)$ disjoint from $\bar{U}$ and the proof is complete. For $a = c - 1$, the complement of $\bar{U}$ is itself convex and intersects every $R(p)$, since $R(p)$, intersecting every $T(q)$, cannot be a subset of $\bar{U}$ by hypothesis. There is then a closed convex set disjoint from $\bar{U}$ which intersects every $R(p)$, and the proof is complete.

We remark that for the case of a single commodity on each side, Theorems 1A and 3 yield $v(Z, 1, t\delta) \to 1, 0$ as $t \to \infty$ according as the value of the game with matrix $H = \| h(u, v) = z_2(u, v) - z_1(u, v) \|$ is positive or negative.

3. EXAMPLES

Example 1. $a = 2$, $c = 4$,

$$Z = \begin{Vmatrix} (0, 0, 1, 0) & (1, 0, 0, 0) \\ (0, 1, 0, 0) & (0, 0, 0, 1) \end{Vmatrix}.$$

This is the game in which I's commodities are women and cats, II's are men and mice, and each engagement is a duel between one of I's units (chosen by him) and one of II's units (chosen by him), with the rule woman defeats man, who defeats cat, who defeats mouse, who defeats woman. If A, B, C, D are the initial supplies of women, cats, men, mice respectively, the matrix in terms of units with initial supplies unity is

$$Z_1 = \begin{Vmatrix} (0, 0, c, 0) & (a, 0, 0, 0) \\ (0, b, 0, 0) & (0, 0, 0, d) \end{Vmatrix}$$

where $aA = bB = cC = dD = 1$. From Theorem 1B, I wins if we can find two numbers α, β, $0 \le \alpha, \beta \le 1$ for which the set of $z = (z_1, \ldots, z_4)$ for which $z_1 < \alpha z_3 + (1 - \alpha) z_4$, $z_2 < \beta z_3 + (1 - \beta) z_4$ intersects every $T(q)$, $0 \le q \le 1$ where $T(q)$ is the line segment joining $(a(1 - q), 0, cq, 0)$ and $(0, bq, 0, d(1 - q))$. A straightforward but tedious computation shows that α, β exist if and only if $ab < cd$, in which case we may take $\alpha = \beta = \frac{1}{2}$. This conclusion can easily be verified directly. The point P on $T(q)$ with $z_1 = z_2$ is given by $(bq + a(1 - q))P = (abq(1 - q), bcq^2, ad(1 - q)^2)$. The required inequalities become

$$2abq(1 - q) < bcq^2 + ad(1 - q)^2,$$

which is easily seen to hold for all q if and only if $ab < cd$, i.e. $AB > CD$. From the symmetry of the problem, II wins if $AB < CD$.

Example 2. $a = 2$, $c = 3$,

$$Z = \begin{Vmatrix} (0, 12, 10) & (9, 0, 10) \\ (0, 9, 10) & (12, 0, 10) \end{Vmatrix}$$

The purpose of this example is to indicate the nature of the winning strategies (see [1] for a description of the winning strategies in general); specifically, this is an example in which, for initial supplies of all commodities equal and large, I is certain to win, but has no winning strategy of the form $\{f_n = p\}$.

The set $T(q)$ is the line segment joining $P_1 = (9-9q, 12q, 10)$ and $P_2 = (12-12q, 9q, 10)$. Since $U = \{\max(z_1, z_2) < z_3\}$ is convex and $P_1 \in U$ for $q < \frac{5}{6}$, $P_2 \in U$ for $q > \frac{1}{6}$, the hypotheses of Theorem 1 are satisfied and I wins (with probability near 1 for large equal initial supplies). Suppose, however, that I uses a strategy of the form $\{f_n = p\}$, i.e. on each play chooses row 1 with probability p, regardless of the outcome of previous plays. If II chooses column 1 in each play, the average loss vector per play is $(0, 12p + 9(1 - p), 10)$, so that, if $p > \frac{1}{3}$, I's average loss in commodity 2 exceeds 10 and he will almost certainly lose in a large number of plays. Similarly, for $p < \frac{2}{3}$, I will almost certainly lose if II chooses column 2 in each play. Thus I's winning strategy cannot be of the form $\{f_n = p\}$.

Actually, I has a winning strategy which is pure, i.e. does not involve randomization, and which is certain to win if the initial supplies are sufficiently large (not merely with probability approaching 1 as the initial supplies become infinite). This winning strategy is: if the past losses in commodity 1 exceed those in commodity 2, choose row 1; if not choose row 2. To see that this strategy wins, denote the losses in commodities 1 and 2 during the first N plays by a_N, b_N respectively, so that $a_N + b_N \leq 12N$. We must show that $\max(a_N, b_N) < 10N$ for sufficiently large N, no matter what strategy II uses. We shall show by induction on N that

$$\max(a_N, b_N) \leq 9N + 3$$

for all N, with actual inequality for $N > 1$. This clearly holds for $N = 1$. Assume the inequality for a given $N \geq 1$. If $a_N \geq 9N$ then $a_N > b_N$ so that,

$$a_{N+1} \leq a_N + 9 \leq 9(N + 1) + 3.$$

If $a_N < 9N$, then $a_{N+1} < 9N + 12 = 9(N + 1) + 3$. Thus in either case $a_{N+1} < 9(N + 1) + 3$. Similarly $b_{N+1} < 9(N + 1) + 3$, completing the proof. The given strategy wins whenever the conflict requires at least 3 plays, i.e. for equal initial supplies of more than 21 units of each commodity.

4. REMARKS

(1) It may happen that the outcome of an engagement depends on chance as well as on the strategies of the two players, so that each element of the matrix Z is a probability distribution over a bounded subset of c-space. All the results of section 2 can be extended to this case without difficulty, simply by considering instead of Z the matrix $\bar{Z}$ of mean values of the elements of Z.

(2) We conjecture that the criterion of Theorem 1 is essentially necessary without the restriction to $a = 1$ or $c = 1$, i.e. for every A, a, either $\bar{U}$ has a convex subset intersecting every $T(q)$ or $\overline{eU}$ has a convex subset intersecting every $R(p)$.

(3) For fixed Z, a, the function $v(Z, a, z) = f(z)$ satisfies the equation

$$f(z) = \text{value of game with matrix } \| h(u, v) = f[z - z(u, v)] \|,$$

so that our results may be interpreted as statements about the asymptotic behavior of solutions of equations of this type. For instance Example 1 of Section 3 has

$$f(A, B, C, D) = \frac{\gamma\delta - \alpha\beta}{\gamma + \delta - \alpha - \beta},$$

where $\alpha = f(A - 1, B, C, D)$, $\beta = f(A, B - 1, C, D)$, $\gamma = f(A, B, C - 1, D)$, and $\delta = f(A, B, C, D - 1)$, with initial conditions $f(A, B, C, D) = 1$ if $AB > 0$, $CD = 0$, $f(A, B, C, D) = 0$ if $AB = 0$, $CD > 0$. Our result is that $f(tA, tB, tC, tD) \to 1$ or 0 as $t \to \infty$ according as $AB >$ or $< CD$.

REFERENCE

[1] David Blackwell, "An analog of the minimax theorem for vector payoffs." Submitted to the Pacific Journal of Mathematics.

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MINIMIZING THE NUMBER OF TANKERS TO MEET A FIXED SCHEDULE

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It is shown that the problem of determining the minimum number of tankers required to meet a fixed schedule of transportation of Navy fuel oil can be made into a linear programming problem of the transportation type. The simplex algorithm is then applied to solve a particular scheduling problem.

1. INTRODUCTION

C. Tompkins (see [3]) has given a discrete idealization of a scheduling problem which arose in the routing of Navy fuel oil tankers. A combinatorial problem of this kind has also been discussed by J. Robinson and J. Walsh [2], together with a proposed method of computation. The algorithm they outline, however, fails to narrow the number of possibilities sufficiently to make it a feasible computational method for most problems.

In this note we show how the tanker scheduling problem can be made into a linear programming problem of transportation type, albeit large. The size of the system is mitigated somewhat by the following facts: (1) most of the variables are constrained to be zero, (2) the minimizing form is particularly simple, and (3) even a large transportation type problem having no special features can be solved by hand using the simplex algorithm [1].

2. THE PROBLEM

A rectangular array of spaces is furnished, one row for each pickup point and one column for each discharge point. In each space (i, j), $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ is a sequence of numbers t_{ij}^k, $k = 1, 2, \ldots$ representing the times at which a tanker is to load fully at pickup point i to deliver to destination j. For example, in the array

	1	2	3
1	1, 4, 7, 10, 13	9, 15	6, 12
2	3, 6, 9, 12	7, 10, 13, 15	5, 10, 15

the sequence 3, 6, 9, 12 in box (2, 1) means that at these times a tanker is to begin loading at pickup point 2 for delivery to discharge point 1. Multiple loads to go from i to j can be taken care of by repetitions in the sequence t_{ij}^k. We make the further assumption, not made in [3], that the total number of entries in the table $t = (t_{ij}^k)$ is finite.

In addition, two arrays of positive numbers a_{ij} and b_{ij} are given, where a_{ij} represents the loading-traveling time from i to j, and b_{ij} the unloading-traveling time from j to i.

The problem is to rearrange the numbers t_{ij}^k into s sequences such that

(2.1) each sequence is monotone increasing;

(2.2) if $t_{i_1j_1}^{k_1} < t_{i_2j_2}^{k_2}$ are consecutive numbers in any one of the s sequences, then

$$t_{i_2j_2}^{k_2} - t_{i_1j_1}^{k_1} \geq a_{i_1j_1} + b_{i_2j_1};$$

(2.3) s is minimal.

In other words, each sequence is a schedule for one tanker, and the objective is to meet the fixed schedule given by table t with a minimum number of tankers.

If, in the example, we take

$$a_{ij} = b_{ij} = \begin{array}{|c|c|c|} \hline 2 & 3 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array},$$

then a feasible schedule using seven tankers would be represented by the rearrangement

(1)	$t_{11}^1 = 1,\ t_{23}^1 = 5,\ t_{22}^1 = 7,\ t_{11}^5 = 13$
(2)	$t_{21}^1 = 3,\ t_{21}^2 = 6,\ t_{21}^3 = 9,\ t_{21}^4 = 12,\ t_{12}^2 = 15$
(3)	$t_{11}^2 = 4,\ t_{12}^1 = 9,\ t_{22}^4 = 15$
(4)	$t_{13}^1 = 6,\ t_{22}^2 = 10,\ t_{23}^3 = 15$
(5)	$t_{11}^3 = 7,\ t_{23}^2 = 10,\ t_{22}^3 = 13$
(6)	$t_{13}^2 = 12$
(7)	$t_{11}^4 = 10.$

3. REFORMULATION AS A PROGRAMMING PROBLEM

For convenience in exposition, we suppose that the numbers t_{ij}^k, a_{ij}, b_{ij} are positive integers. It will be clear that this is not essential to the method of solution.

First construct the table of sequences $T = (t_{ij}^k + a_{ij})$. Thus, T_{ij}^k are the times when tankers loaded at i will arrive at j. Define

$n_{\alpha i}$ = number of times $t_{ij}^k = \alpha$ occurs in row i of t;

$N_{\beta j}$ = number of times $T_{ij}^k = \beta$ occurs in column j of T;

i.e., $n_{\alpha i}$ is the number of tankers loading at i at time α and $N_{\beta j}$ is the number arriving at j at time β. Thus, $n_{\alpha i}$ is defined for $\alpha = 1, 2, \ldots, \max t_{ij}^k$ and $N_{\beta j}$ is defined for $\beta = 1, 2, \ldots, \max T_{ij}^k$.

For any schedule, denote the number of reassignments from discharge point j at time β to loading point i at time α by $x_{\alpha i \beta j}$. More precisely, $x_{\alpha i \beta j}$ is the number of occurrences in a schedule of consecutive pairs $(t^k_{i'j}, t^\ell_{ij'})$ with $T^k_{i'j} = \beta$, $t^\ell_{ij'} = \alpha$. Then for all possible schedules, the inequalities

$$\sum_{\alpha, i} x_{\alpha i \beta j} \leq N_{\beta j}$$

(3.1)

$$\sum_{\beta, j} x_{\alpha i \beta j} \leq n_{\alpha i}$$

(3.2) $$x_{\alpha i \beta j} \geq 0$$

are satisfied. In addition, it follows from (2.1) and (2.2) that

(3.3) $$b_{ij} > \alpha - \beta \text{ implies } x_{\alpha i \beta j} = 0.$$

The system (3.1) can be made into a system of equalities which is formally of transportation type by introducing non-negative slack variables $x_{\alpha i}$, $y_{\beta j}$ and $z = \sum_{\alpha, i} \sum_{\beta, j} x_{\alpha i \beta j}$. Then (3.1) may be rewritten as

$$\begin{aligned}
&\sum_{\alpha, i} x_{\alpha i \beta j} + y_{\beta j} = N_{\beta j}, \qquad y_{\beta j} \geq 0 \\
&\sum_{\beta, j} x_{\alpha i \beta j} + x_{\alpha i} = n_{\alpha i}, \qquad x_{\alpha i} \geq 0 \\
&\sum_{\alpha, i} x_{\alpha i} + z = \sum_{\alpha, i} n_{\alpha i}, \qquad z \geq 0 \\
&\sum_{\beta, j} y_{\beta j} + z = \sum_{\beta, j} N_{\beta j},
\end{aligned}$$

(3.4)

and hence each schedule leads to an integral solution of (3.2) and (3.4) which satisfies condition (3.3).

Conversely, given any integral solution of (3.2), (3.3), and (3.4), a schedule can be constructed from it as follows: Each $x_{\alpha i}$ will be the number of tankers which start their individual schedules at time α from loading point i; i.e., there will be $x_{\alpha i}$ sequences in the rearrangement which have a $t^k_{ij} = \alpha$ as first member. Delete one such $t^{k_o}_{i_o j_o} = \alpha_o$ from t; let $\beta_o = T^{k_o}_{i_o j_o} = \alpha_o + a_{i_o j_o}$. Since $N_{\beta_o j_o} > 0$, at least one of the variables $x_{\alpha i \beta_o j_o}$, $y_{\beta_o j_o}$ has a positive value. Select one such.

Case 1. $x_{\alpha_1 i_1 \beta_o j_o} > 0$ was selected. Then there is a $t^{k_1}_{i_1 j_1} = \alpha_1$, since $n_{\alpha_1 i_1} > 0$. Assign α_1 as second member of the sequence. Observe that by (3.3), $\alpha_1 - \beta_o \geq b_{i_1 j_o}$, hence

$$t^{k_1}_{i_1 j_1} - t^{k_o}_{i_o j_o} = \alpha_1 - \beta_o + a_{i_o j_o} \geq a_{i_o j_o} + b_{i_1 j_o}$$

and (2.2) is satisfied. Strike out α_1 from t and reduce $x_{\alpha_1 i_1 \beta_o j_o}$, $N_{\beta_o j_o}$, $n_{\alpha_1 i_1}$ by unity.

Case 2. $y_{\beta_o j_o} > 0$ was selected. In this case the sequence ends with α_o. Reduce $y_{\beta_o j_o}$, $N_{\beta_o j_o}$ by unity.

If Case 1 obtained, let $\beta_1 = T^{k_1}_{i_1 j_1} = \alpha_1 + a_{i_1 j_1}$, and examine the values of the variables $x_{\alpha i \beta_1 j_1}$, $y_{\beta_1 j_1}$. Again one of these must be positive. Apply either Case 1 or Case 2 with α_1 playing the role of α_o. Repetition of the procedure outlined must eventually end with the selection of some $y_{\beta_k j_k} > 0$ (since by (3.3), $\alpha \leq \beta$ implies $x_{\alpha i \beta j} = 0$), thus completing one of the sequences. The others can be gotten in the same way.

Notice that while many schedules can be constructed from an integral solution of (3.2), (3.3), and (3.4), the only physical difference between two such is that there may be more than one tanker available at the same time at the same pickup point or discharge point, in which case they may be interchanged without affecting the total number of tankers.

Thus, the tanker scheduling problem can be viewed as one of minimizing $\sum_{\alpha, i} x_{\alpha i}$, the number of sequences in a rearrangement (or what is the same thing, maximizing the variable z), over the set of integral solutions[1] of (3.2) and (3.4) in which the variables designated by (3.3) are fixed at zero. But it is well known [1] that the maximum of a linear form defined over all solutions of (3.2) and (3.4) is always assumed at some integral solution, and it is easy to see that this fact is not altered by imposing additional constraints of the form $x_{\alpha i \beta j} = 0$. Hence the scheduling problem can be solved by the simplex algorithm, since the nature of the algorithm is such as to obtain a required integral solution. Moreover, the algorithm is extremely simple to apply when the problem is of transportation type, as is this one.

It is obvious also that linear programming can be used to optimize schedules with respect to other costs. For example, it would be simply a matter of changing the minimizing form, holding z fixed, to find a schedule for a given number of tankers which has the least sailing time.

4. A NUMERICAL EXAMPLE

We continue with the example of Section 2. First form the table of arrival times

$T =$

	1	2	3
1	3, 6, 9, 12, 15	12, 18	8, 14
2	4, 7, 10, 13	9, 12, 15, 17	6, 11, 16

Using this table and the one of loading times, compute all $n_{\alpha i}$ and $N_{\beta j}$. After discarding those rows and columns having $n_{\alpha i}$ or $N_{\beta j}$ zero, one is left with the transportation problem whose constraints are indicated schematically in Fig. 1. Crossed out cells mean that the corresponding variable is constrained to be zero by (3.3). The solution shown in Fig. 1 corresponds to the schedule using seven tankers given in Section 2. This is a degenerate solution to the programming problem and so it is necessary in applying the simplex algorithm to pick out other basic

[1] Such whole number solutions always exist, e.g., take all $x_{\alpha i \beta j} = 0$. This corresponds to the worst possible schedule of assigning a different tanker for each trip.

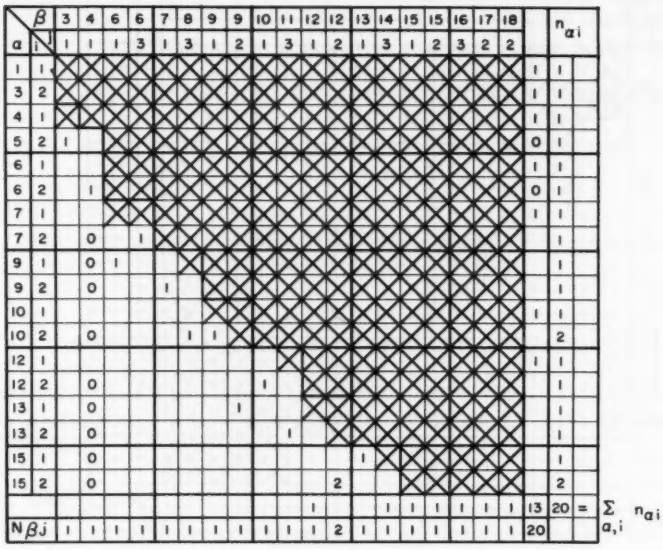

Figure 1

variables having zero values. One way of doing this is shown in Fig. 1 by the placement of the O's.

An optimal solution, reached after a few iterations, is shown in Fig. 2. It corresponds to the following six-tanker schedule:

(1)	$t_{11}^{1} = 1,\ t_{23}^{1} = 5,\ t_{22}^{1} = 7,\ t_{11}^{5} = 13$
(2)	$t_{21}^{1} = 3,\ t_{21}^{2} = 6,\ t_{21}^{3} = 9,\ t_{13}^{2} = 12$
(3)	$t_{11}^{2} = 4,\ t_{12}^{1} = 9,\ t_{22}^{4} = 15$
(4)	$t_{13}^{1} = 6,\ t_{22}^{2} = 10,\ t_{23}^{3} = 15$
(5)	$t_{11}^{3} = 7,\ t_{23}^{2} = 10,\ t_{21}^{4} = 12,\ t_{12}^{2} = 15$
(6)	$t_{11}^{4} = 10,\ t_{22}^{3} = 13.$

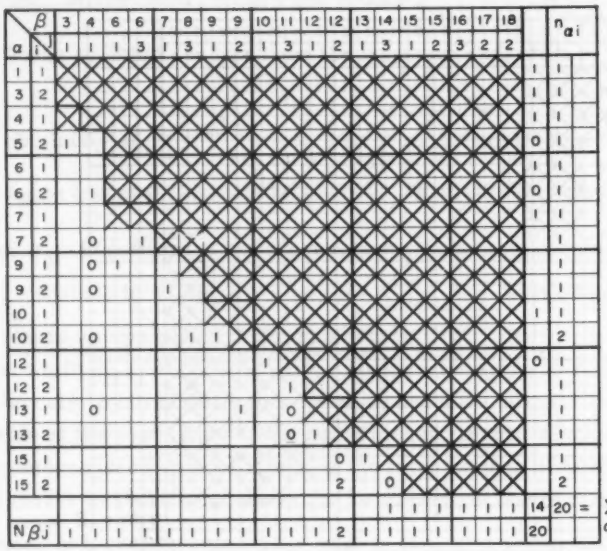

Figure 2

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ON THE SOLUTION OF THE CATERER PROBLEM[1]

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In the June 1954 issue of this Quarterly, Walter Jacobs has shown how a problem arising in connection with the overhaul of engines by the Air Force could be treated by linear programming. This paper presents an explicit algorithm for computing the solutions of the problem.

1. INTRODUCTION

This paper continues the investigation of the Caterer problem formulated and partially solved by Walter Jacobs in [1]. We shall describe and justify an algorithm for solving the problem, indeed for solving a problem that is slightly more general than the original one. A knowledge of [1] is desirable for an interpretation of the abstractly formulated linear program discussed here, but is not necessary for a reading of this paper. We believe, however, that the reader who is interested in the "life cycle" of a linear program - (a) formulation, (b) simplification, (c) development of an algorithm for solution - will learn from the discussion of the caterer problem contained in [1] (which treats (a), (b) and a special case of (c)) and this article an instructive, and in some ways, typical example.

We now introduce the problem to be attacked:

Let $t_1, \ldots, t_m$ be given constants, $r_1, \ldots, r_n$ given non-negative constants, let X be a variable, $z = (z_1, \ldots, z_n)$ be a variable vector lying in the rectangular parallelopiped

$$0 \leqq z_i \leqq r_i \qquad (i = 1, \ldots, n). \tag{1}$$

Let $A_1, \ldots, A_m$ be given vectors in n-space, the coordinates of each consisting of 1's and 0's such that:

(2) The 1's in each A_i are consecutive, and

(3) If the coordinates in A_i which are 1 are $a, a+1, \ldots, b$, and the coordinates in A_j which are 1 are $c, c+1, \ldots, d$, and if $a < c$, then $b \leqq d$. We assume tacitly that each A_i contains at least one 1.

The problem is: for any number $\lambda \geqq 0$, to find values for X and z, subject to (1) and

$$(A_i, z) \geqq t_i - X \qquad (i = 1, \ldots, m) \tag{4}$$

which minimize the linear form

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(5) $$\lambda X + \sum_{i=1}^{n} z_i .$$

A comparison of our problem with the transformed caterer problem given in (4.3) of [1] will show that (4.3) is a special case of the above.

2. THE SOLUTION IF X IS SPECIFIED

Since our object is to study the behavior of X and z, it is not unnatural to ask what the solution of our problem would be if X were specified; i.e., if X were a given constant. If we set $\mu_i = t_i - X$ $(i = 1, \ldots, m)$, then our problem becomes: for all $z = (z_1, \ldots, z_n)$ subject to (1) and

(4') $$(A_i, z) \geq \mu_i \qquad (i = 1, \ldots, m),$$

minimize

(5') $$z_1 + \ldots + z_n.$$

Before solving the problem, one should check to see that the inequalities (1) and (4') are consistent. Set $r = (r_1, \ldots, r_n)$. Then it is clear that the inequalities are consistent if and only if $(A_i, r) \geq \mu_i$ $(i = 1, \ldots, m)$, which can be easily ascertained.

Assume now that (1) and (4') are consistent. Let S_j be the set of all indices i such that the j-th coordinate of A_i is 1. Define $r^{(0)} = (0, r_2, r_3, \ldots, r_n)$, and

(6) $$z_1(X) = \max \left\{ 0; \max_{i \in S_1} [\mu_i - (A_i, r^{(0)})] \right\}.$$

If $z_1(X), \ldots, z_k(X)$ have been defined $(1 \leq k \leq n)$, let $r^{(k)} = (z_1(X), \ldots, z_k(X), 0, r_{k+2}, \ldots, r_n)$. Then define

(7) $$z_{k+1}(X) = \max \left\{ 0; \max_{i \in S_{k+1}} [\mu_i - (A_i, r^{(k)})] \right\}.$$

We contend that the vector $z(X) = (z_1(X), \ldots, z_n(X))$ defined inductively above solves the problem of this section.

It is clear that our prescription states that $z_1(X)$ is the smallest value of z, consistent with (1) and (4'); and that, assuming $z_1 = z_1(X), \ldots, z_k = z_k(X)$, $z_{k+1}(X)$ is the smallest value of z_{k+1} consistent with (1) and (4').

We shall prove our contention by showing that

LEMMA 1: For every k $(k = 1, \ldots, n)$, $z(X)$ minimizes $z_1 + \ldots + z_k$, subject to (1) and (4').

PROOF BY INDUCTION: We have already seen that Lemma 1 holds if $k = 1$. Assume it holds for $k - 1$, we wish to show it holds for k. If $z_k(X) = 0$, this is obvious from the induction hypothesis. If $z_k(X) \neq 0$, then by (7), there exists a vector A_i such that

(8) $$z_k(X) + (A_i, r^{(k-1)}) = u_i, \; i \in S_k.$$

Let us write out the inequality

(9) $$(A_i, z) \geq \mu_i$$

as

(9') $$z_{k-\alpha} + \ldots + z_{k-1} + z_k + z_{k+1} + \ldots + z_{k+\beta} \geq \mu_i.$$

(Recall (2). Of course α or β or both may be 0.) Then (8) says that

$$z_{k-\alpha}(X) + \ldots + z_k(X) + r_{k+1} + \ldots + r_{k+\beta} = \mu_1. \tag{10}$$

Now (9') is one of the inequalities (4'). Hence, by (10) and (1), $z_{k-\alpha}(X) + \ldots + z_k(X) = \min(z_{k-\alpha} + \ldots + z_k)$, for all z subject to (1) and (4'). But by the induction hypothesis, $z_1(X) + \ldots + z_{k-\alpha-1}(X) = \min(z_1 + \ldots + z_{k-\alpha-1})$. Combining these last two statements proves the induction step.

Note that our proof uses (2), but not (3). The lemma is false if (2) does not hold.

Now, it is clear from (4) that the set of X for which the inequalities (1) and (4) of our main problem are consistent form a closed half-line $\mathfrak{L}$ containing arbitrarily large positive numbers. Let e be the end point of $\mathfrak{L}$. For each $X \geq e$, let $S(X) = \min(z_1 + \ldots + z_n)$, subject to (1) and (4). We have just seen how to compute S(X) and we now prove

LEMMA 2: For $X \geq e$, S(X) is a convex non-increasing function of X.

PROOF: Let $X_1 > X_2$; then the convex region which is the intersection of the half-spaces corresponding to $X = X_1$ in (4) contains the convex region corresponding to $X = X_2$; hence $S(X_1) \leq S(X_2)$. To prove S(X) is convex, we first prove a more general lemma.

Let A and B be two real linear spaces, and let K be a convex set in $A \times B$. Let V be the projection of K on A; i.e., $V = \{v \mid v \in A, \exists\ w \in B \text{ such that } v \times w \in K\}$; let W be the projection of K on B. It is easy to see that V and W are convex sets. Let f be a convex function bounded from below defined on V.

LEMMA 3: The function

$$g(w) = \inf_{v \times w \in K} f(v) \tag{11}$$

is a convex function on W.

PROOF: We must show that $w_1 \in W$, $w_2 \in W$, $\alpha \geq 0, \beta \geq 0, \alpha + \beta = 1$, imply

$$g(\alpha w_1 + \beta w_2) \leq \alpha g(w_1) + \beta g(w_2). \tag{12}$$

Let ϵ be any positive number. Then by (11), there exist v_1, v_2 such that

$$v_1 \times w_1 \in K, \quad v_2 \times w_2 \in K, \text{ and} \tag{13}$$

$$f(v_1) \leq g(w_1) + \epsilon, \quad f(v_2) \leq g(w_2) + \epsilon. \tag{14}$$

Since K is convex, it follows from (13) that

$$\alpha(v_1 \times w_1) + \beta(v_2 \times w_2) = (\alpha v_1 + \beta v_2) \times (\alpha w_1 + \beta w_2) \in K.$$

Therefore, since f is convex,

$$g(\alpha w_1 + \beta w_2) \leq f(\alpha v_1 + \beta v_2) \leq \alpha f(v_1) + \beta f(v_2) \leq \alpha g(w_1) + \beta g(w_2) + \epsilon,$$

by (14). Since this inequality holds for every $\epsilon > 0$, we have (12).

Apply Lemma 3 to the present situation as follows: Let $z = v$, $z_1 + \ldots + z_n = f(v)$, $X = w$, $S(X) = g(w)$. This completes the proof of Lemma 2.

3. SUMMARY OF THE SITUATION TO DATE

Let us assume, temporarily that S(X) is explicitly known. Now it is manifest from (6) and (7) that $z_j(X)$ is a polygonal curve. Hence $S(X) = \sum_{j=1}^{n} z_j(X)$ is a polygonal curve. Since

$S(X)$ is convex, the half-line $\mathcal{L}$ will break up into intervals $(e = e_o, e_1), (e_1, e_2), \ldots, (e_d, \infty)$, such that the slope of $S(X)$ is $-m_1$ in (e_o, e_1), $-m_2$ in $(e_1, e_2), \ldots, -m_d$ in (e_{d-1}, e_d), 0 in (e_d, ∞), $m_1 > m_2 > \ldots > m_d > 0$. It is obvious that $e_d = \max_i t_i$ and that in (e_d, ∞), $S(X) = 0$. It is also clear that $e_o = \max_i \left\{t_i - (A_i, r)\right\}$. Assuming that all e_δ and m_δ were known, it is obvious that the minimum of (5) (which, as the sum of two convex polygonal functions of X, is also a convex polygonal function of X) could be obtained as follows:

if $\lambda \geq m_1$, the minimum of (5) is attained at $X = e_o$,

if $m_1 \geq \lambda \geq m_2$, the minimum of (5) is attained at $X = e_1$,

. . .

. . .

. . .

(15) if $m_{d-1} \geq \lambda \geq m_d$, the minimum of (5) is attained at $X = e_{d-1}$,

if $m_d \geq \lambda \geq 0$, the minimum of (5) is attained at $X = e_d$,

if $0 > \lambda$, (5) has no minimum.

Since we have already seen in Section 2 how to obtain the z_j which minimize (5) once the correct X is known, formulas (15) solve our problem once the numbers $e_o, e_1, \ldots, e_d$ and $m_1, \ldots, m_d$ are determined.

We already know e_o. We shall show below how, given $e_{\delta-1}$ one may obtain e_δ and m_δ by a reasonably efficient procedure. Since e_d is also known, this procedure, in view of (15), will solve our problem.

4. THE ALGORITHM

Recall now the procedure described in paragraph 2, particularly, formulas (6) and (7). All that we need do, in view of what has already been stated, is to keep track of each $z_j(X)$ as X varies. The remainder of the paper is devoted to showing how to do this in an expeditious way.

It is suggested that the reader examine the example given in Section 5 as he reads the description given in this section.

An inequality $(A_i, z) \geq t_i - X$ is said to be bounding for z_j at X if $i \in S_j$ and $z_j(X) = \mu_i - (A_i, r^{(j-1)})$. Let B_j be the set of all indices i such that $(A_i, z) \geq t_i - X$ is bounding for z_j at $X = e_{\delta-1}$. Note that

(16) $$z_j(e_{\delta-1}) > 0$$

implies that B_j is not empty. Let T_i be the set of all indices j such that the j-th coordinate of A_i is 1. Then

(17) $$i \in B_j,\ k > j,\ k \in T_i \text{ imply } i \in B_k,\ z_k(e_{\delta-1}) = r_k.$$

We now divide all indices $i = 1, \ldots, n$ into two classes M (moving) and S (stationary) as follows: if $z_1(e_{\delta-1}) = 0$, assign 1 to S; if $z_1(e_{\delta-1}) > 0$, assign 1 to M; if $1, 2, \ldots, k$ have been assigned, and $z_{k+1}(e_{\delta-1}) = 0$, assign $k+1$ to S; if $z_{k+1}(e_{\delta-1}) > 0$, and no index in $\bigcup_{i \in B_{k+1}} T_i$ (recall (16)) has been assigned to M, we assign $k+1$ to M, but if an index in $\bigcup_{i \in B_{k+1}} T_i$ has been assigned to M, we put $k+1$ in S.

It will turn out that m_i is the number of indices assigned to **M**, but before we show this, we first prove

LEMMA 4: (a) If $i \in \bigcup_j B_j$, then there is at most one index in the intersection of T_i and **M**; (b) for any i, there are at most two indices in the intersection of T_i and **M**.

PROOF OF (a): Assume $i \in B_j$; $k, \ell \in T_i$, $k, \ell \in M$, $k < \ell$. If $\ell \geq j$, then (17) and the definition of **M** contradict $\ell \in M$. Therefore, we may assume $k < \ell < j$.

Let $i_o \in B_\ell$ (we know from (16) that $\ell \in M$ implies B_ℓ is not empty). Then since $k \in M$, $k < \ell$, it follows that $k \notin T_{i_o}$. But $k, \ell, j \in T_i$; hence by (2) and (3), $p > \ell$, $p \in T_i$ imply $p \in T_{i_o}$. But by (17), $p \in T_{i_o}$, $p > \ell$ imply $z_p(e_{\delta-1}) = r_p$. Together with $i \in B_j$, these facts imply $i \in B_\ell$. We know, however, that $k \in M$, $k \in T_i$. Combined with $\ell \in T_i$, we have a contradiction of $\ell \in M$.

PROOF OF (b): Assume $j < k < \ell$; $j, k, \ell \in T_i$, $j, k, \ell \in M$. Let $p \in B_k$. Then $j \notin T_p$, and also, by (17), $\ell \notin T_p$. A comparison of T_i and T_p exhibits a violation of (2) and (3).

We now resume our main discussion on how to go from $e_{\delta-1}$ to e_δ. Let **K** be the set of all i such that the intersection of T_i and **M** consists of two indices (**K** may be empty). Let $\beta = \min_{i \in K} \left\{ (A_i, z(e_{\delta-1})) - t_i + e_{\delta-1} \right\}$ if **K** is not empty, $\beta = +\infty$ otherwise. Then $\beta > 0$. For if $\beta = 0$, i.e., there is an $i_o \in K$ such that $(A_{i_o}, z(e_{\delta-1})) = t_i - e_{\delta-1}$, then if k is the largest index in T_{i_o}, clearly $i_o \in B_k$, and we would have a violation of Lemma 4 (a).

Let $\gamma = \min_{i \in M} z_i(e_{\delta-1})$. By the definition of **M**, $\gamma > 0$. Finally, let $\alpha = \min(\beta, \gamma) > 0$. We contend

LEMMA 5: For all X, $e_{\delta-1} \leqq X \leqq \alpha + e_{\delta-1}$ and each $j = 1, \ldots, n$, we have

$$z_j(X) = \overline{z_j(X)} = \begin{cases} z_j(e_{\delta-1}) & \text{if } j \in S \\ z_j(e_{\delta-1}) - (X - e_{\delta-1}) & \text{if } j \in M \end{cases} \tag{18}$$

Before proving the lemma, let us informally state its content. If X lies in the prescribed range, then: if $j \in S$, $z_j(X) = z_j(e_{\delta-1})$; i.e., $z_j(X)$ is stationary. If $j \in M$, $z_j(X)$ moves by precisely the amount by which X differs from $e_{\delta-1}$.

PROOF: Obviously the lemma holds for $z_1(X)$. Assuming it holds for $z_1, \ldots, z_{k-1}(X)$, we shall show it holds for $z_k(X)$. By virtue of Lemma 1, we must show that if $z_1 = z_1(X), \ldots, z_{k-1} = z_{k-1}(X)$ satisfy (18), $z_{k+1} = r_{k+1}, \ldots, z_n = r_n$, then (i) if we set $z_k = \overline{z_k(X)}$ according to (18), the vector z satisfies (1) and (4); but (ii) if we set $z_k < \overline{z_k(X)}$ the vector z will not satisfy (1) and (4).

Suppose $k \in M$. We first prove (i). For each $i \in S_k$, T_i may contain no indices in the range $1, \ldots, k-1$ in **M** or 1 index in the range $1, \ldots, k-1$ in **M** (Lemma 4 (b)). It is clear from the induction hypothesis that, in the former case, $(A_i, z) \geq t_i - X$. In the latter case, again from the induction hypothesis, we have

$$(A_i, z) \geq (A_i, z(e_{\delta-1})) - 2(X - e_{\delta-1}). \tag{19}$$

But, by the definition of α and β, for this range of X,

$$X - e_{\delta-1} \leq (A_i, z(e_{\delta-1})) - t_i + e_{\delta-1}. \tag{20}$$

Combining (19) and (20), we obtain again $(A_i, z) \geq t_i - X$. Next, let $i \in B_k$. Then clearly $(A_i, z) = t_i - X$, which proves (ii).

Suppose $k \in S$. The proof of (i) involves no new ideas, and is omitted. To prove (ii), if $z_k(e_{\delta-1}) = 0$, (ii) is immediate; hence, assume $z_k(e_{\delta-1}) > 0$, which, by (16), (17), and the construction of S, implies that there exists an $i \in B_k$, $j \in S_i$, $j < k$ such that $j \in M$. It follows from (17) and (7) that $(A_i, z) = t_i - X$, which proves (ii).

We observe the following immediate consequences of Lemma 5.

COROLLARY 1: The function $z_j(X)$ is a monotonic, non-increasing function of X.

This implies that in going from $e_{\delta-1}$ to e_δ, we may substitute for (1) the inequalities

$$0 \leq z_j(X) \leq z_j(e_{\delta-1}). \tag{1'}$$

COROLLARY 2: The slope m_1 is equal to the number of indices in M.

To complete our algorithm, when we arrive at $X = \alpha + e_{\delta-1}$ we replace $e_{\delta-1}$ by $e_{\delta-1} + \alpha$ and repeat our construction of the new M and S, say M' and S'. By virtue of the convexity of S(X), the number of indices in M' is less than or equal to the number of indices in M. If it is less than, then $\alpha + e_{\delta-1} = e_\delta$. If it is equal, we proceed in the same manner. It is clear, from the fact that each $z_j(x)$ is a polygonal function of X with a finite number of vertices, and (from the definition of α) that each step takes us from an X corresponding to a vertex of some $z_j(X)$ to the next largest X corresponding to a vertex of (possibly some other) $z_j(X)$, that we arrive at e_δ in a finite number of steps.

5. AN EXAMPLE

Let us assume that the given system of inequalities is

$$(*)\ z_1 + z_2 + z_3 \geq 10 - X$$

$$(**)\ z_1 + z_2 \geq 7 - X \qquad 0 \leq z_i \leq 5\ (i = 1, 2, 3).$$

$$(***)\ z_2 + z_3 \geq 9 - X$$

We wish to minimize $z_1 + z_2 + z_3 + \lambda X$.

Then

$$e_o = \max(10-15, 7-10, 9-10) = -1$$

$$e_d = \max(10, 7, 9) = 10.$$

The graphs of $z_1(X)$, $z_2(X)$, $z_3(X)$ and $S(X)$ are shown in Figure 1.

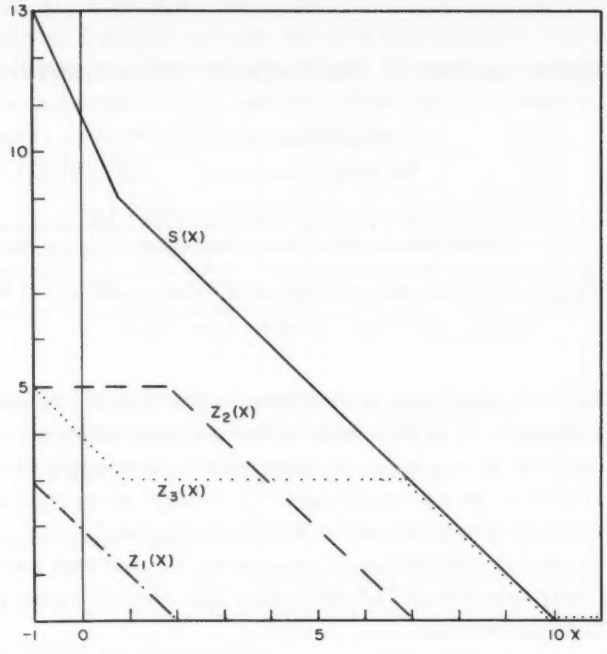

Figure 1

The work sheet, following the prescriptions of (5) and (6), is as follows:

	$z_1(X)$	$z_2(X)$	$z_3(X)$	B_1	B_2	B_3	M	S	β	γ	α
$X = -1 = e_0$	3	5	5	(**)	(**), (***)	(***)	1, 3	2	2	3	2
$X = 1 = e_1$	1	5	3	(**)	(**)	(*), (***)	1	2, 3	∞	1	1
$X = 2$	0	5	3	(**)	(**)	(*)	2	1, 3	∞	5	5
$X = 7$	0	0	3	empty	empty	(*)	3	1, 2	∞	3	3
$X = 10 = e_2 = e_d$	0	0	0								

Thus, the solutions to our problem are:

	X	z_1	z_2	z_3
$\lambda \geq 2$	-1	3	5	5
$2 \geq \lambda \geq 1$	1	1	5	3
$1 \geq \lambda \geq 0$	10	0	0	0

REFERENCE

[1] Jacobs, W., "The Caterer Problem," Naval Research Logistics Quarterly, Vol. 1 (1954), pp. 154-165.

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DECISION MAKING IN THE FACE OF UNCERTAINTY—I

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It is shown in this paper that for a number of simple multi-stage decision processes the intuitive concept of maximizing expected gain over expected cost at each stage is approximately correct.

1. INTRODUCTION

In logistics there is a large class of situations in which we are to use a given resource repeatedly until it is exhausted. If at each stage we have several different ways of utilizing this resource, it is a problem of importance to determine the procedure which maximizes the over-all value of the resource. In a previous paper [1] we have given some applications of the theory of dynamic programming to problems of this general class.

In this paper we wish to consider some representative problems where the outcome of any stage is not completely determined. We shall show that in some cases optimal policies of quite simple and intuitive nature exist.

In subsequent papers we shall discuss some cases of uncertain purpose, which is to say, processes in which we do not completely know the form of the "payoff" or criterion function.

2. DISCUSSION

A problem of recurrent type is the following: We have a certain quantity of resource which we can use in various fashions. Any particular application yields a certain distribution of returns balanced against a certain distribution of costs. How do we proceed to use the resource until depleted so as to maximize the average return?

A naive approach to the problem runs as follows: As a result of any particular decision we obtain a certain expected gain and suffer a certain expected cost. A reasonable procedure for maximizing over-all gain would then seem to be one which yields the maximum return per unit cost, i.e., one which maximizes the ratio

$$R = \frac{\text{immediate expected gain}}{\text{immediate expected cost}}. \tag{2.1}$$

This prescription for an optimal policy has many desirable features. It is simple, intuitive, requires only an estimation of average values rather than detailed knowledge of the distribution of events—and is occasionally correct.

We shall discuss below a number of situations in which it is approximately correct.

The purpose of the simple models discussed below is to bolster our intuition, which is, after all, only a combination of the results of theory and practice. As far as the multi-stage processes of realistic type are concerned, existent theory is meager. Consequently, it is important to build up a backlog of as many mathematical models as possible with known solutions in order to obtain clues to the realistic problems which usually defy precise analysis.

3. A MULTI-STAGE ALLOCATION PROBLEM OF DETERMINISTIC TYPE

Let us begin with a simpler process which is deterministic. We have an initial resource x which may be utilized in a number of ways. If y is a parameter specifying a particular use, we let R (x, y) be the immediate return, and D (x, y) the cost in resources. How do we proceed to utilize this resource so as to maximize the total return?

We set (see [1], [2], [3])

(3.1) $$f(x) = \text{total return from an initial resource } x, \text{ using an optimal allocation policy.}$$

Then, as discussed in [1], [2], this function satisfies the functional equation

(3.2) $$f(x) = \operatorname{Max}_y [R(x, y) + f(x - D(x, y))].$$

Let us now make the fundamental assumption that D (x, y) is small compared to x for any y, i.e. $0 < D(x, y) << x$. Proceeding formally, we may write

(3.3) $$f(x) = \operatorname{Max}_y [R(x, y) + f(x) - D(x, y) f'(x) + \ldots]$$

which yields the approximate equation

(3.4) $$0 = \operatorname{Max}_y [R(x, y) - D(x, y) f'(x)].$$

This means that for one y, say $\bar{y}$, we have

(3.5) $$0 = R(x, \bar{y}) - D(x, \bar{y}) f'(x),$$

and for all others we have

(3.6) $$0 \geq R(x, y) - D(x, y) f'(x).$$

Consequently, within the error contained in using (3.4) in place of (3.3), we have

(3.7) $$f'(x) = \operatorname{Max}_y \frac{R(x, y)}{D(x, y)}$$

which is equivalent to the statement that at each stage we use our resource in accord with the prescription of (2.1).

4. MULTI-STAGE ALLOCATION—STOCHASTIC

Let us now consider the process above where we have a distribution of returns and allocations. Any particular utilization yields a set of returns, z, characterized by a distribution function R (y, z, x), and a set of costs, w, characterized by a distribution function D (y, w, x). We now wish to maximize the expected total return from an initial resource x. Denote this quantity by f (x). Then, as above, f (x) satisfies the equation

(4.1) $$f(x) = \operatorname{Max}_y [\int_0^\infty z dR(y, z, x) + f(x - \int_0^\infty w dD(y, w, x))].$$

Assuming, as before, that

(4.2) $$\text{Expected Cost} = \int_0^\infty w dD(y, w, x) << x,$$

we obtain as an approximation to (4.1), the equation

$$0 = \max_y \left[\int_0^\infty z\,dR(y,z,x) - f'(x) \int_0^\infty w\,dD(y,w,x) \right], \tag{4.3}$$

or

$$f'(x) = \max_y \frac{\int_0^\infty z\,dR}{\int_0^\infty w\,dD} \tag{4.4}$$

$$= \max_y \frac{\text{Immediate Expected Return}}{\text{Immediate Expected Cost}}$$

which is precisely the prescription of (2.1).

5. CONCLUSION

We have discussed above two representative examples of multi-stage allocation processes, in each of which the prescription of (2.1) furnished an approximation to the optimal policy.

In those situations where the change in resources may be large compared to initial resources, the above analysis is not as useful. Nonetheless, even in these cases, this prescription furnishes a useful first approximation which may be improved by successive approximations based upon the functional equations (see [1], [2], [3]).

REFERENCES

[1] Bellman, R., "On Some Applications of the Theory of Dynamic Programming to Logistics," Naval Research Logistics Quarterly, Vol. 1 (1954), pp. 141-153.

[2] Bellman, R., "Some Problems in the Theory of Dynamic Programming," Econometrica, Vol. 22 (1954), pp. 37-48.

[3] Bellman, R., "Some Applications of the Theory of Dynamic Programming—A Review," Journal of the Operations Research Society of America, Vol. 2 (1954), pp. 275-288.

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NEWS AND MEMORANDA

Readers are invited to submit to the Managing Editor items of general interest in the field of logistics.

On September 1, 1954, Dr. Edward W. Cannon, the Principal Investigator of The George Washington University Logistics Research Project, returned to the National Bureau of Standards to resume his duties as Assistant Chief of the Applied Mathematics Division. He has also become the Chief of the newly formed Mathematical Physics Section of that Division. Dr. Max A. Woodbury is the new Principal Investigator of The George Washington University Logistics Research Project.

On June 30, 1954 the Institute for Numerical Analysis at the University of California at Los Angeles, a section of the Applied Mathematics Division of the National Bureau of Standards, was discontinued. Many of the activities formerly assigned to the Institute for Numerical Analysis are being administered by the University of California under the sponsorship of several agencies of the Department of Defense. Most of the recent members of the Institute for Numerical Analysis, including the director, Dr. C. B. Tompkins, continued under the new arrangement. The new name and address of this activity is Numerical Analysis Research, University of California, 405 Hilgard Avenue, Los Angeles 24, California.

The George Washington University Logistics Research Project's Fifth Annual Logistics Research Conference has been set for the week beginning October 25, 1954. The mission of the Conference is to develop effective plans for the utilization of usage data by the Navy. The first phase of the Conference will consist of an issue of "Logistics Papers" presenting some background material, and of papers to be delivered during the first two days of the Conference. In the second phase, the conferees will participate in six separate panels meeting on assigned subjects. The reports of the individual panels will be presented during the third and final phase.

A symposium on Linear Programming is to be held in Washington, D. C. during January 27-29, 1955, under the joint sponsorship of the U. S. Air Force and the National Bureau of Standards. There will be sessions on practical applications, computation procedures, economics, and the theory of linear inequalities. The meetings are open, and there will be no registration fee. Additional information concerning the Symposium can be obtained from Dr. Alan J. Hoffman, Applied Mathematics Division, National Bureau of Standards, Washington 25, D. C.

The Social Science Research Council plans to hold two 8-week sessions on mathematics for social scientists during the summer of 1955, one at the University of Michigan and the other at Stanford University. These sessions will be open to predoctoral and postdoctoral students

and younger faculty members in the social sciences who wish to improve their mathematical competence. Copies of a detailed announcement can be obtained from the Social Science Research Council, 726 Jackson Place, N. W., Washington 6, D. C.

St. John's College and the American Military Institute will sponsor a seminar on the philosophy of war in the Washington, D. C. area. Meetings will be held one evening each week for about 20 weeks beginning on October 19, 1954. Further information can be obtained from The Office of Adult Education, St. John's College, Annapolis, Maryland.

The RAND Corporation has undertaken a major research program in the area of Air Force logistics. RAND, since its inception in 1946, has done research almost exclusively for the Air Force. Particular emphasis in the past has been placed on weapons system analysis, strategy, and problems of Air Force policy and economics. Although logistics aspects of many specific problems have been analyzed as a part of other studies, it was not until the latter half of 1953 that RAND was able to devote a major research effort to logistics per se.

As now developed, the Logistics Section is in the Economics Division, and consists of twenty-one professional people. The Section is under the direction of Stephen Enke, one of the early members of the Economics Division and formerly a Professor of Economics at UCLA. Murray Geisler, who formerly worked in the Air Comptroller's Office in the Pentagon under General Huglin, is in charge of the Logistics Systems Group within the Logistics Section. Russell B. McNeill is in charge of the Procedures Group; he was formerly a member of the faculty and an Assistant Dean at the Graduate School of Business Administration at Harvard University. Allen R. Ferguson, formerly on the faculty at the University of Virginia, and recently a government economist and economic consultant, serves as Dr. Enke's assistant and is in charge of research in Transportation.

Other members of the Section include Alan S. Manne, formerly an Assistant Professor of Economics at Harvard University, and Bruse Moncreiff, who has done research on electronic data processing systems with Battelle Memorial Institute.

The Section will have access to the full resources of the RAND Corporation, including close intellectual ties particularly with the Economics, Mathematics, and Aircraft Divisions, and the use of the Numerical Analysis staff and equipment, including RAND's IBM 701.

The Section has developed a program of research which in the near future emphasizes three major problems: the introduction of electronic data processing and communication equipment into the Air Force supply system; the rationalization of Air Force inventory policy; and the introduction of air transportation for routine resupply. It is expected that the program will be expanded in the future to include other areas of Air Force logistics.

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RECENT PUBLICATIONS

THEORY OF GAMES AND STATISTICAL DECISIONS. By David Blackwell and M. A. Girshick, John Wiley & Sons, Inc., 1954, 355 pp.

This book is intended as a textbook on statistical decision theory, which as developed by A. Wald during the period from 1939 to 1950, includes all statistical problems. It begins with a treatment of those parts of the theory of games of importance in statistical decision theory. In order to avoid measure-theoretic difficulties, the formal development treats only discrete distributions, but continuous distributions are sometimes used for illustrative purposes. The chapter headings are as follows: Games in Normal Form, Values and Optimal Strategies in Games, General Structure of Statistical Games, Utility and Principles of Choice, Classes of Optimal Strategies, Fixed Sample-Size Games with Finite Ω, Fixed Sample-Size Games with Finite A, Sufficient Statistics and the Invariance Principle in Statistical Games, Sequential Games, Bayes and Minimax Sequential Procedures When Both Ω and A Are Finite, Estimation, and Comparison of Experiments.

ECONOMICS OF NATIONAL SECURITY. By George A. Lincoln and associates, 2nd Edition, Prentice-Hall, Inc., 1954, 643 pp.

The second edition is a complete revision of the 1950 edition and is intended to be of interest to businessmen, government officials, and civilian leaders. Current and future problems have been emphasized over the historical approach of the previous edition. The book has been rearranged into four sections to facilitate its use by those interested in only one or two of the sections. The section headings are as follows: I. National Security - The Problem, II. Resources and Their Mobilization, III. Finance and Stabilization, and IV. International Aspects. The book is a corporate effort of the members of the Department of Social Sciences, U. S. Military Academy. On controversial subjects the authors have presented counterarguments along with their personal views so as to provide a basis for further discussion. No attempt has been made to document all facts and views, many of which were obtained from public officials and government publications.

ECONOMIC ACTIVITY ANALYSIS. Edited by Oskar Morgenstern, John Wiley & Sons, Inc., 1954, 554 pp.

In this volume are collected studies made by the Economics Research Project at Princeton University. All the 12 contributors are members of the Project or were associated with it at one time or another, and the 13 papers are closely related to each other. The volume

is divided into three parts: 1. Economic Properties of the Input-Output System, 2. Mathematical Properties of Linear Economic Systems, and 3. Meta-economics. The papers range in difficulty from very simple, descriptive, numerical analyses to fairly difficult mathematical investigations. The contributors are: J. B. Balderston, C. Bernstein, R. Bott, O. Eckstein, J. P. Mayberry, K. Menger, C. P. Modlin, Jr., O. Morgenstern, G. Rosenbluth, T. M. Whitin, Y. K. Wong, and M. A. Woodbury.

THE FOUNDATIONS OF STATISTICS. By Leonard J. Savage, John Wiley & Sons, Inc., 1954, 294 pp.

This book presents the author's opinions on the foundations of statistics. He brings out the difficulty of accepting the objectivistic views on probability, as is done my most mathematical statisticians in this country, and attempts to replace it by a personalistic view as expressed by Bruno de Finetti. The book consists of seventeen chapters, the first seven of which are concerned with the basic foundations and its application to the behavior of an individual making decisions under uncertainty. In the last ten chapters, the author treats certain topics in statistics in accordance with the views developed in the first part of the book.

THE COMPLEAT STRATEGYST. By J. D. Williams with pictorial illustrations by Charles Satterfield, McGraw-Hill Book Co., Inc., 1954, 234 pp.

This book deals with the problems of the development and application of the theory of games of strategy. It is restricted to the two-person game with several strategies for each side. Numerous practical illustrations are given; some military, some economic, and others of general occurrence in human activity. The book is strictly elementary, requiring only arithmetic; many exercises are provided, enabling the reader to tackle significant problems. The relation of game theory to linear programming is also discussed.

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